Math 123
Fall 2023
Dr. Lily Yen

Midterm II

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Name: _____
Number: _____
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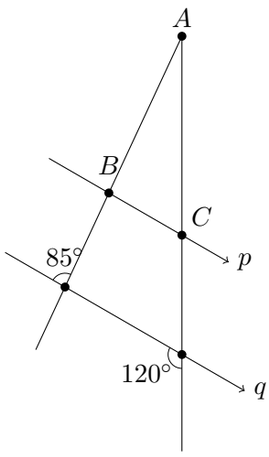
Score: ___/34

Problem 1: Lines p and q are parallel. Find the measures of INTERIOR angles in ΔABC:

$\angle A =$ 25°, $\angle B =$ 95°, and $\angle C =$ 60°.



The given angle 85° is corresponding to the exterior angle at B, so the interior angle, $\angle B = 180° - 85° = 95°$. Similarly, 120° is a corresponding angle to the exterior angle at C, so $\angle C = 180° - 120° = 60°$ form a straight line, so they add up to 180°. The last angle, namely, $\angle A$ is found by using triangle angle sum, so $\angle A = 180° - 95° - 60° = 25°$.

Score: /3

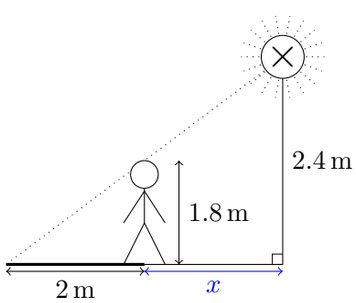
Problem 2: Set up a table for all rectangles with area 12 cm^2 whose dimensions are integral in centimetre(s). Of these rectangles, which one has the longest perimeter?

Rectangle:	
Perimeter:	

The 1×12 rectangle has the longest perimeter of 26 cm. The other rectangles are 2×6 with a perimeter of 16 cm , and 3×4 with a perimeter of 14 cm and their 90° rotations.

Score: /3

Problem 3: Shown is a 1.8 m tall pedestrian standing near a lamp post of 2.4 m high. If the pedestrian's shadow is 2 m long, how far away is the pedestrian standing away from the base of the lamp post?

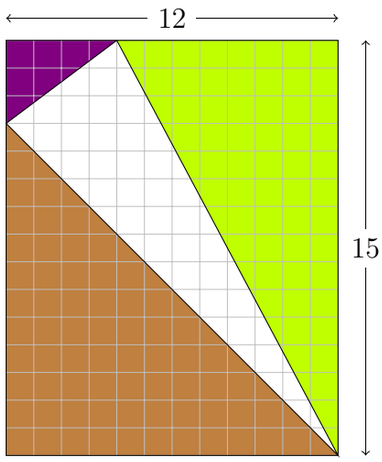


If the distance from the pedestrian to the lamp post is x, then, by similar triangles, $\frac{x+2}{2} = \frac{2.4}{1.8}$, so $x + 2 = 2 \times \frac{2.4}{1.8} = \frac{8}{3}$, so $x = \frac{8}{3} - 2 \approx 0.67$ m.

Score: /3

/9

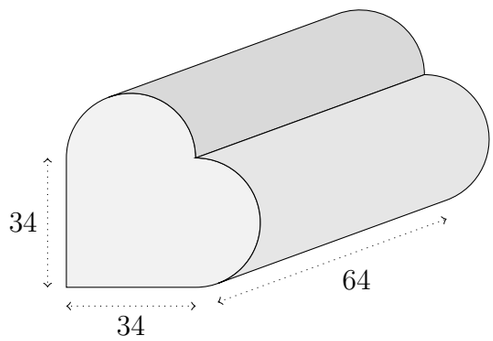
Problem 4: Below is a 12×15 grid containing a narrow triangle lying diagonally. Find the exact (no decimal approximation) perimeter of the narrow triangle. Show steps for each side of the triangle.



By Pythagorean Theorem, the shortest side has length 5 (from the 3, 4, 5 triangle), another side is the hypothenuse of an 8, 15, 17 triangle, thus side length 17. The third side is an isosceles right triangle of side 12, so the hypothenuse is $12\sqrt{2}$. All together, the perimeter is $22 + 12\sqrt{2}$.
Since the simplification of a square root is not expected, $22 + \sqrt{288}$ is also accepted.

Score: /4

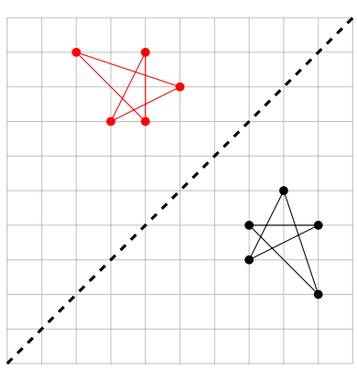
Problem 5: Euclid put two half cylinders on a square prism to make a heart prism. Find its volume.



The volume is $34^2 \times 64 + \pi 17^2 \times 64 = 73\,984 + 18\,496\pi \approx 132\,091$ cubic units.

Score: /4

Problem 6: Reflect the given figure along the given diagonal.



Score: /2

/10

Problem 7: Brian's second brother, Patrick, bought some NFT for $300 in 2021. After a year, in order to raise money for an emergency, he sold it for $285. What was the rate of his loss in percent?

Patrick lost 5 % of the value by computing $(285 - 300)/300$, or the rate was $-5\,\%$.

Score: /2

Problem 8: How much would Brian's grandparents need to put in a GIC earning 1.5 % to have a million dollars in 30 years? Use simple interest.

$1\,000\,000 = P(1 + 0.015 \times 30)$, so $P = 1\,000\,000/1.45 \approx 689\,655.17$ dollars.
Some used compound interest compounded annually instead of the simple interest formula. If so, the answer would be 639 762.43 dollars. Since this is a harder question, no mark is deducted if the steps and the final answer are correct.

Score: /3

Problem 9: Bank of Mom and Pop offered children under 18 years old 3-year GICs worth $1500 when matured for every $1200 dollars invested. Suppose the GIC was compounded quarterly, what was the rate Bank of Mom and Pop offered?

If $1500 = 1200(1 + r/4)^{4\times 3}$, thenr $1.25 = (1 + r/4)^{12}$. Raise both sides to the fractional power of $1/12$ to get $(1.25)^{1/12} = 1 + r/4$. Subtracting 1 on both sides,

$$\frac{r}{4} = (1.25)^{1/12} - 1, \quad \text{so} \quad r = 4 \times ((1.25)^{1/12} - 1) \approx 0.075\,077\,060\,5,$$

around 7.5 %.

Score: /3

/8

Problem 10: David's friend, Jason, graduated from university with a student loan of \$30 000 at 4 % to be paid off in 5 years.

a. What would the monthly instalment be using the add-on interest method?

b. How much was the total finance charge on the loan?

c. After the first instalment, what was the balance on the loan?

a. The monthly instalment would be $30\,000 \times (1 + 0.04 \times 5)/(12 \times 5) = 600$, using add-on interest method.

b. The total finance charge on the loan would be $30\,000 \times 0.04 \times 5 = 6000$ dollars.

c. The balance on the loan after the first instalment can be found after finding the interest portion of the first instalment of \$600. Since \$6000 in interest was spread over 60 months, the first month's interest was \$100, so \$500 went to reducing the loan from \$30 000 to \$29 500, the new balance on the loan.

Score: /5

Problem 11: If *Capilano Express* credit card charges 22 % on unpaid balance, how much would it cost in finance charge to leave \$570 unpaid past the due date for 90 days?

$I = Prt = 570 \times 0.22 \times \frac{90}{365} \approx 30.92$ dollars.

Score: /2

/7