Math 123
Fall 2023
Dr. Lily Yen

Midterm I

Show all your work

Name: _____
Number: _____
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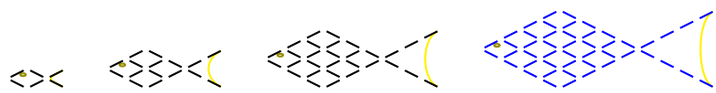
Score: ___/30

Problem 1: Anjali used toothpicks to form a sequence of fish.



uses 48 toothpicks.

a. Draw the fourth fish in the sequence.

b. Count the total number of toothpicks in the fourth fish.

c. Establish a table of pattern to obtain a formula involving n to express the total number of toothpicks in the nth fish.

Fish	1	2	3	4	...	n
Toothpicks	6	16	30	48	...	$2n(n+1)+2n$

Score: /4

Problem 2: The sum of Katharina's parents' ages is 118. Given that her father is two years older than her mother, find their ages.

Let x be Katharina's mother's age and y be her father's age. The sum is $x + y = 118$, and the age difference is $x + 2 = y$. Therefore, when y in the first equation is replaced by $x + 2$, one obtains $x + x + 2 = 118$, or $x = 58$, and $y = 60$.
Thus, Katharina's father is 60 and her mother is 58 years old.
Other approaches are, for example, trial and error, or a table of patterns, or starting with $118 \div 2 = 59$, then adjust one up (for dad) and one down (for mom).

Score: /2

Problem 3: Lena-Anna is a Michelin-Starred restaurant with tables which seat 4 or 2. Suppose there are 5 more tables which are 2-seaters than 4-seaters. At capacity, the restaurant serves 40 people. How many tables of each type does Lena-Anna have?

Let x be the number of 4-seater tables, then $2(5 + x) + 4x = 40$, or $6x = 30$. So, $x = 5$.
There are 5 tables seating 4 people each, and 10 tables seating 2 people each.

Score: /2

Problem 4: Mei rolls a six-sided die, an eight-sided die, and a twenty-sided die simultaneously. How many possible outcomes are there? Use a tree diagram to support your answer.

Mei can expect $6 \times 8 \times 20 = 960$ outcomes.

Score: /2

/10

Problem 5: Fire Horse is looking at the following from different numeration systems. Convert each to a Hindu-Arabic numeral, and perform indicated operations.

a. 

b. 

c. MCDLXIX

d. Perform the subtraction $1010011_2 - 11011_2$ in binary, then convert each binary including the answer to base-10 to check.

e. Convert $A3B_{16}$ to an octal number.

f. 

a. $35 \times 60^2 + 12 \times 60 + 47 = 126\,767$.

b. $18 \times 20^2 + 6 \times 20 + 12 = 7332$.

c. $1000 + (500 - 100) + 50 + 10 + (10 - 1) = 1469$.

d. Convert first binary to $64 + 16 + 2 + 1 = 83$. Convert the second binary to $16 + 8 + 2 + 1 = 27$. Since $83 - 27 = 56$, we check that the binary answer is $32 + 16 + 8 = 56$.

$$\begin{array}{r} 1010011_2 \\ -\quad 11011_2 \\ \hline 111000_2 \end{array}$$

e. $10 \times 256 + 3 \times 16 + 11 = 2619$, using successive division by 8, one obtains 5073_8.

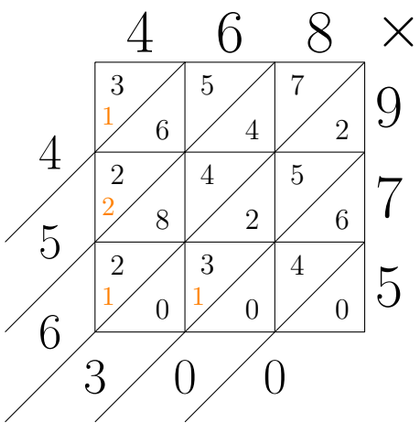
f. $2 \times 18 \times 20 + 12 \times 20 + 13 = 973$.

Score: /11

/11

Problem 6: Multiply 468×975 using the galley method.

456 300



Score: /3

Problem 7: Find three different values for x satisfying $x + 3 \equiv 5 \mod 13$.

List first the multiples of 13 yields: 0, 13, 26, 39, …
Then list the numbers with remainder 5 when divided by 13: 5, 18, 31, 44, …
Since $x + 3 \equiv 5 \mod 13$, $x + 3 = 5$, or $x + 3 = 18$, or $x + 3 = 31$; thus, $x = 2$, or 15, or 28.

Score: /3

Problem 8: A group of students are attending a seminar at the university auditorium. There are chairs arranged to the left and to the right of the podium. Chairs on the left are arranged 7 in a row, and chairs on the right are arranged 10 in a row. If all students sit on the left, from the first row on, without any empty seat, there are exactly 5 students in the last row before empty rows. If all students sit on the right, again filling all seats from the first on, there are exactly 2 students in the last row before empty rows. Find the second smallest number of students that attend the seminar.

If the students siting on the left take up a full rows, then there are $7a + 5$ students in attendance. If the same students fill b full rows on the right, their number is $10b + 2$. Therefore $7a + 5 = 10b + 2$, so $7a + 3 = 10b$. The integer solutions to this equation are

$a =$	1	11	21	…
$b =$	1	8	15	…
Total $10b + 2 =$	12	82	152	…

Score: /3

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