Math 123
Fall 2022
Dr. Lily Yen

Test 1

Show all your work

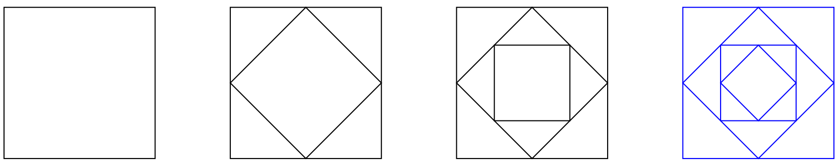
Name: ______________

Number: ______________

Signature: ______________

Score: ___/35

Problem 1: Beginning with a square, Anjali draws another square by joining midpoints of each side to produce the second picture. Continuing in this pattern, she draws the third picture.



a. Draw the fourth picture in the sequence.

b. Count the total number of squares of all sizes in the fourth picture. **4**

c. Count the total number of triangles of all sizes in the fourth picture. **12**

d. Establish a formula $T(n)$ for the total number of triangles in the nth picture.

Figure	1	2	3	$\dots$	n
Triangles	0	4	8		$T(n) = 4(n-1)$

Score: /2

Problem 2: Signing up with Shaw Home Internet, Mei's family was given six free cell-phone numbers with unlimited domestic calls and texts. If their six numbers all have the same area code (778) and the same first three digits (843), how many different choices do they have to fill the last four digits? What if the last 4 digits must be distinct and also distinct from what's already used in the previous 6 spots?

Only four digits are free, so $10^4 = 10\,000$ choices.
Since digits $\{3, 4, 7, 8\}$ are used, only 6 digits are left. To make them all distinct, there are $6 \times 5 \times 4 \times 3 = 360$ choices of the last 4 digits.

Score: /2

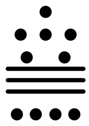
Problem 3: Katharina organized a bag of Haribo's gummy bears according to colour. If Katharina had five times as many red gummy bears as blue ones, and the red and blue gummy bears together equalled the rest of the gummy bears, list all the possible total numbers of gummy bears organized by Katharina, starting with the smallest possible number.

Assume 1 blue gummy bear with 5 red gummy bears and 6 of the rest. The smallest possible number is 12, all multiples of 12, or $12n$ are possible.

Score: /2

Problem 4: Order the following 5 numerals in ascending order, that is, from smallest to largest. One mark is given for each correct conversion into Hindu-Arabic numeral.

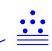
a. 

b. 

c. MMMCDLXIV

d. 

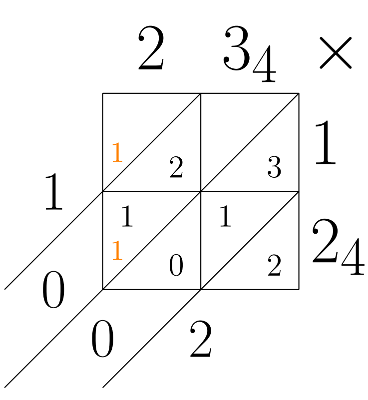
e. $D54_{16}$

 $= 3216 <$  $= 3306 < D54_{16} = 3412 <$ MMMCDLXIV $= 3464 <$  $= 8624$

$a < d < e < c < b.$

Score: /6

Problem 5: Use the Galley Method to multiply $23_4 \times 12_4$ in base-4 leaving the answer in base-4.



Score: /3

Problem 6: When Anjali went to a pumpkin patch with a group of international students, she noticed that when she organized all the pumpkins her group picked into rows of 5 or rows of 7 each, she had 3 left. However, when she organized them 6 in a row, she had 4 left. Find the smallest possible number of pumpkins satisfying the conditions of her arrangements.

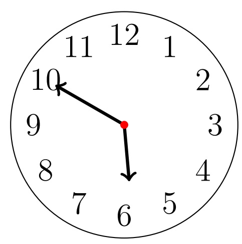
Let p be the number of pumpkins. Since $p-3$ multiple of both 5 and 7, it follows that $p-3$ is a multiple of $\operatorname{lcm}(5,7)=35$, so

$$p = 3 + 35k \quad \text{for some integer } k.$$

Trying $k = 0, 1, 2, \ldots$ the first value of p that leaves remainder 4 when divided by 6 is 178.

Score: /3

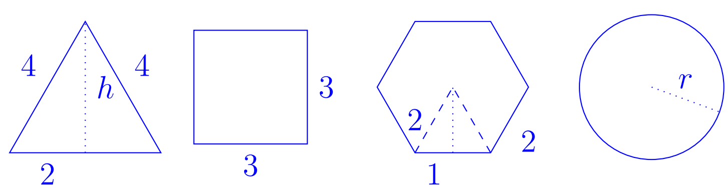
Problem 7: Find the smaller angle formed by the hour and minute hands at ten to six on a twelve-hour analogue clock.



Let North (at 12) be $0°$ position. Then the hour hand at $5:50$ would be at $5 \times 30° + 50°/2 = 150° + 25° = 175°$ position because the hour hand sweeps $0.5°$ per minute. The minute hand at 50 minutes is at $50 \times 6° = 300°$ because it sweeps $6°$ per minute. Therefore, the smaller angle formed by the hands is $300° - 175° = 125°$.

Score: /3

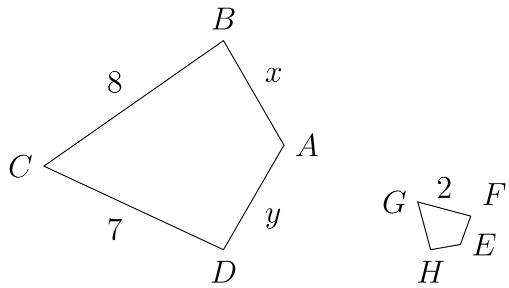
Problem 8: Take 4 ropes of equal length, each 12 units long: use the first to form an equilateral triangle; the second, a square; the third, a regular hexagon; the fourth, a circle. Note that each has the same perimeter of 12 units. Find the area of each shape **except the hexagon**. Conjecture which shape has the greatest area in this construction. Provide some rationale to your conjecture.



By the Pythagorean Theorem, $h^2 + 2^2 = 4^2$, so $h = \sqrt{12}$, so the area of the triangle is $\frac{4h}{2} = 2h = 2\sqrt{12} \approx 6.928$.
The area of the square is $3^2 = 9$.
The triangle inside the hexagon has height $\sqrt{2^2 - 1^2} = \sqrt{3}$, so its area is $\frac{2\sqrt{3}}{2} = \sqrt{3}$. Since the hexagon contains six triangles, its area is $6\sqrt{3} \approx 10.392$.
The circumference of the circle is $2\pi r = 12$, so $r = 6/\pi$, so the area is $\pi r^2 = \pi(6/\pi)^2 = \pi(36/\pi^2) = 36/\pi \approx 11.459$.
The circle has the greatest area since it has no tight corners.

Score: /5

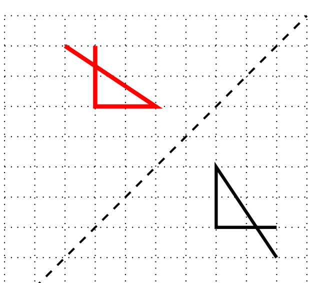
Problem 9: Given two similar quadrilaterals: quadrilateral $ABCD$ is similar to quadrilateral $EFGH$, find the values of x and y given that $\overline{FE} = 1/2$, and $\overline{HE} = 3/5$.



Since quadrilateral $ABCD$ is similar to quadrilateral $EFGH$, $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{EF}}{\overline{FG}}$, so $\frac{x}{8} = \frac{1/2}{2}$, so $x = 8 \times \frac{1}{4} = 2$.
Similarly, $\frac{\overline{DA}}{\overline{HE}} = \frac{\overline{BC}}{\overline{FG}}$, so $\frac{y}{3/5} = \frac{8}{2}$, so $y = \frac{3}{5} \times \frac{8}{2} = \frac{12}{5} = 2.4$.

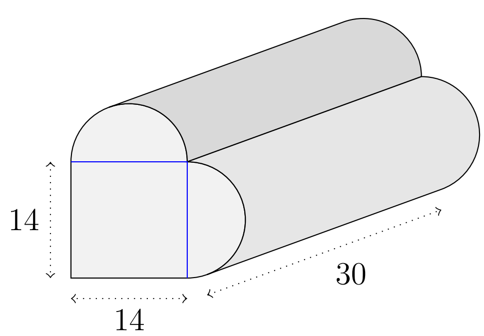
Score: /3

Problem 10: Reflect the given figure along the dashed line.



Score: /2

Problem 11: Find the volume and surface area of the heart shaped prism.



The face towards you consists of a 14×14 square and two semi-circle, each with diameter 14. The area of that face is therefore $14^2 + \pi 7^2 = 196 + 49\pi$. The volume is then $30(196 + 49\pi) = 5880 + 1470\pi \approx 10\,498.1$ cubic units.
The surface consists of the front face found above, a similar face in the back, a 14×30 rectangle at the bottom, another on the left side, and finally the two half-cylinders, each with area $\frac{1}{2} \cdot 14\pi \cdot 30 = 210\pi$. The total surface area in square units is then $2(196 + 49\pi + 14 \cdot 30 + 210\pi) = 1232 + 518\pi \approx 2859.43$.

Score: /4