Math 123
Fall 2022
Dr. Lily Yen

Assignment 1

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/10

Problem 1: In a small group Anjali participated in for New Student Orientation, every student shook hands with one another before the group leader arrived. If the group leader made 10 handshakes by shaking hands with everyone in the group, find the total number of handshakes among the students in Anjali's small group before the leader's arrival.

Since the group leader shook hands with 10 students, we need to find the total number of handshakes among 10 people. This is a sum of consecutive integers from 1 to 9,

$$1 + 2 + 3 + \cdots + 8 + 9 = 45$$

Thus, a total of 45 handshakes among 10 students in Anjali's group.

Score: /2

Problem 2: Below is a 3×3 magic square: using all whole numbers from 1 to 9 exactly once, fill the square such that the row sum, column sum, and diagonal sum are all equal.

2	9	4
7	5	3
6	1	8

We find the sum of the first 9 consecutive positive integers is 45, so $45 \div 3 = 15$, the row sum, column sum, and diagonal sum. The number 5 is the middle number, so should be placed in the middle of the square. The middle column, middle row, and two diagonal sums need to use all four pairs of remaining 8 integers adding to 10 for each pair. When 1 and 9 are placed in one of the diagonals, it is impossible to find four remaining integers such that 1 and two other integers could add to 15 for both the row and the column sums involving 1. This forces 1 and 9 to be in a middle column or a middle row. Once this is done, the other digits can find their places.

Score: /2

Problem 3: Mei has a mason bee house in her garden. Mei's brother collects spiders. If the bees and spiders together have 40 legs, is it possible to have a least one of each? Assume that all bees and spiders have all the legs each is supposed to have: six each for bees and eight each for spiders. The following table may help you organize your work.

If there is no bee, then $5 \times 8 = 40$ would mean that Mei's brother has 5 spiders. However, if one has at least one of each, then the only possible match is to have 4 bees and 2 spiders because $4 \times 6 + 2 \times 8 = 24 + 16 = 40$.

Score: /2

Problem 4: Mei, Katharina, and Anjali went to pick blueberries in Langley. If they picked 33 pounds in total with Anjali picking twice as much as Mei, and three times as much as Katharina, how many pounds did Katharina pick?

Let a denote the number of pounds Anjali picked; m, Mei's; k, Katharina's. We are given $a = 2m$ and $a = 3k$, so $2m = 3k$ or $m = 3k/2$.
Since we know the total,

$$a + m + k = 3k + 3k/2 + k = 11k/2 = 33, \quad \text{so} \quad k = 6.$$

Katharina picked 6 pounds of blueberries.

Score: /2

Problem 5: Determine first the largest number of tries one needs to break a code for a 6-digit lock. What if the digits must be distinct? Provide reasons for each answer.

Using a tree diagram, one can see that each node has 10 branches, thus 10^6 different codes.
With six distinct digits, one has $10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151\,200$.

Score: /2