Math 123
Fall 2018
Dr. Lily Yen

Test 1

Show all your work

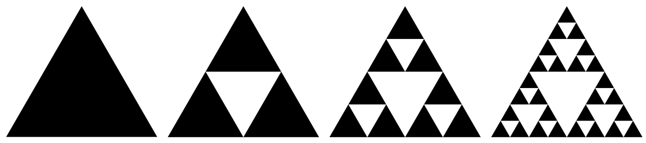
Name: ____________________

Number: ____________________

Signature: ____________________

Score: ___/40

Problem 1: Sierpinski triangle is a fractal: first take an equilateral triangle (stage 0), then draw a middle triangle (stage 1). Below shows the first three stages.



Tabulate the number of *black* triangles at each stage, and conclude with a formula giving the number of black triangles at the n-th stage.

Stage	0	1	2	3	...	n
Black triangles	1	3	9	27		3^n

Score: /2

Problem 2: Suppose you need a new phone number where the area code (first three digits) is fixed, and the last four digits of the ten-digit phone number is the same as your landline number. How many choices do you have for the remaining digits?

Only three digits are free, so $10^3 = 1000$ choices.

Score: /2

Problem 3: Capilano University (initially, Capilano College) was established fifty years ago. Write the year of its birth in Roman numerals.

MCMLXVIII

Score: /2

Problem 4: You are buying treats consisting of M&M's, Skittles, and Coffee Crisps to give out for Halloween. Suppose you have twice as many M&M's as Skittles, and 30 more Coffee Crisps than M&M's, how many of each would you have knowing that you have 200 treats all together?

If you have x Skittles, then you have $2x$ M&M's, and $2x + 30$ Coffee Crisps. All together you have $x + 2x + 2x + 30 = 5x + 30$ treats, so $5x + 30 = 200$, so $5x = 170$, so $x = 34$. You therefore have 34 Skittles, 68 M&M's, and 98 Coffee Crisps.

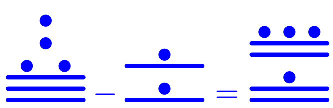
Score: /3

Problem 5: Express 321456 in Babylonian numeral.

Since $321456 = 216000 + 29 \times 3600 + 17 \times 60 + 36$, the Babylonian numeral is

Score: /3

Problem 6: Express the subtraction $397 - 126$ in Mayan numeral, then find its answer in Mayan numeral.



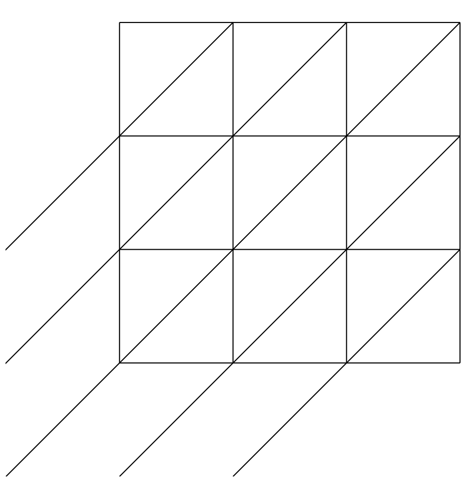
Score: /3

Problem 7: Convert 73 in base 10 to a binary numeral.

In base 10 we have that $73 = 64 + 8 + 1 = 2^6 + 2^3 + 2^0$. Therefore $73_{10} = 1001001_2$.

Score: /2

Problem 8: Multiply 258×476 using the galley method.



122 808

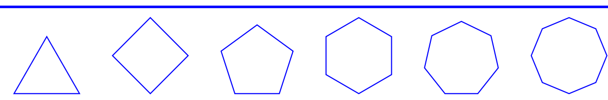
Score: /2

Problem 9: You have a chime clock and a cuckoo clock in the living room. The chime clock chimes every quarter of an hour while the cuckoo clock cuckoos every half hour. Starting at twenty past eleven one morning when our lecture ends, how many times would both clocks announce the time together by three in the afternoon that day? Assume, of course, that the clocks are synchronized.

at 11:30, 12:00, 12:30, 1:00, 1:30, 2:00, 2:30, 3:00. Total 8 times.

Score: /2

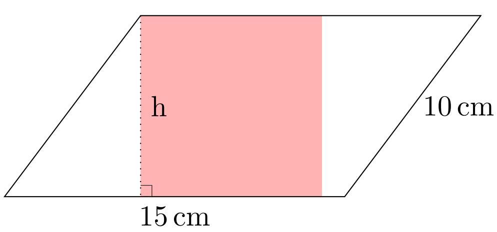
Problem 10: Set up a table for convex polygons' angle sums beginning with a triangle, followed by a quadrilateral, a pentagon, and so on. From your table, derive a formula for the measure of an interior angle in a regular n-sided polygon.

Polygon:								n-gon
Angle sum:	180	360	540	720	900	1080	...	$180(n-2)$

So an interior angle has $180(n-2)/n$ degrees.

Score: /3

Problem 11: Given that the area of the parallelogram shown is $120\,\text{cm}^2$, find the area of the largest square you can draw sharing the height, h of the parallelogram such that all four vertices of the square are on the perimeter of the parallelogram. Include units.



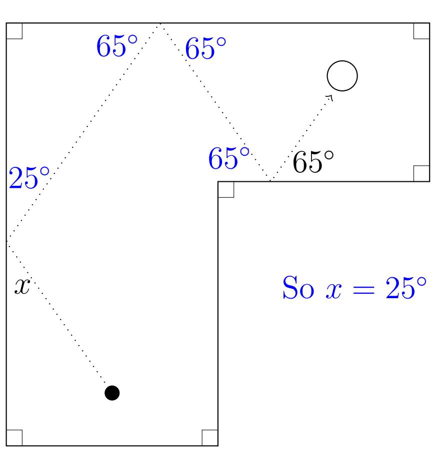
The area of a parallelogram is $A = bh = 15\,\text{cm} \times h = 120\,\text{cm}^2$. So $h = 8\,\text{cm}$. It follows now from the Pythagorean Theorem that $x^2 + h^2 = 10^2$, so $x^2 = 10^2 - h^2 = 36$, so $x = 6\,\text{cm}$. Therefore $y = 15\,\text{cm} - 6\,\text{cm}$, and you do have room for a square with side $8\,\text{cm}$. The area of such a square is $64\,\text{cm}^2$.

Score: /3

Problem 12: Find the measure of angle x to make a hole-in-one at the miniature golf course hole. Use the following two facts to find x:

a. The angle the ball makes as it hits a flat surface has the same measure as the angle the ball makes as it leaves the same surface.

b. The interior angle sum of a triangle is 180°.

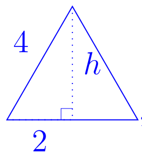


So $x = 25°$

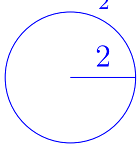
Starting at 65° using condition (a), alternate interior angles of parallel lines, and complementary angles of a right triangle, we reach $x = 25°$.

Score: /3

Problem 13: Which of the two shapes cover more area? An equilateral triangle of side length 4 or a circle of diameter 4. Show your work to support your claim.

In the equilateral triangle 

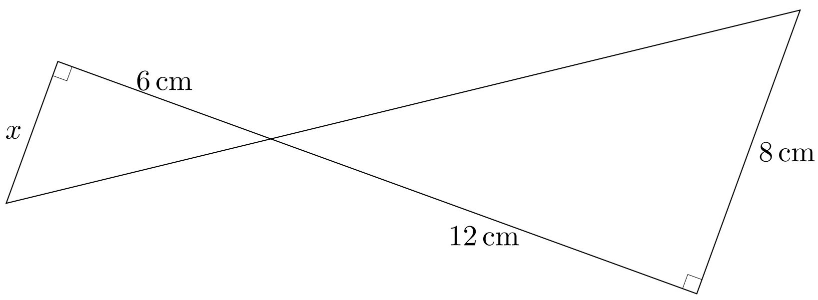
, $h^2 + 2^2 = 4^2$, so $h^2 = 12$, so $h = \sqrt{12} = 2\sqrt{3}$. The area is therefore $\frac{4h}{2} = 2h = 4\sqrt{3} \approx 6.93$.

The circle 

has area $4\pi \approx 12.6$, so it covers almost twice the area of the triangle.

Score: /3

Problem 14: Find x = 4 cm. Name the triangles and provide reasons for your claim.



The two inner angles are vertically opposite so equal. Since each triangle has a right angle, the third angles have to be equal, too. Hence the triangles are similar and $\frac{x}{6} = \frac{8}{12}$, so $x = 6 \times \frac{8}{12} = 4$.

Score: /2

Problem 15: Take two rectangular prisms with the same volume of $24\,\text{cm}^3$. Suppose that the dimensions of both prisms are integer centimetre lengths.

a. Give two examples of such rectangular prisms with different surface areas.

All the possible boxes are

Dimensions	$1 \times 1 \times 24$	$1 \times 2 \times 12$	$1 \times 3 \times 8$	$1 \times 4 \times 6$	$2 \times 2 \times 6$	$2 \times 3 \times 4$
Surface Area	98	76	70	68	56	52

Each dimension is in centimetre, and the surface areas are in square centimetres.

Score: /2

b. Find two such rectangular prisms whose surface areas differ as much as possible.

With a complete listing of volume $24\,\text{cm}^3$ rectangular prisms with integral dimensions, we choose the first one with $98\,\text{cm}^2$ and the last one with $52\,\text{cm}^2$ to achieve the greatest difference in surface area of $46\,\text{cm}^2$.

Score: /3