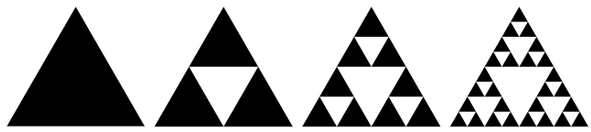
Math 123
Fall 2018
Dr. Lily Yen

Assignment 1

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/16

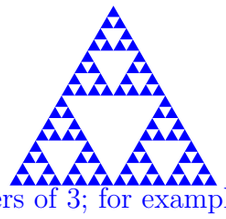
Problem 1: Sierpinski triangle is a fractal: first take an equilateral triangle (stage 0), then draw a middle triangle (stage 1). Below shows the first three stages.



Count the number of *white* triangles (of all sizes) for each stage, and predict according to your pattern the number of *white* triangles of all sizes in the fourth stage.

Stage:	0	1	2	3	4
White triangles:	0	1	4	13	40

$t_n = 3t_{n-1} + 1$



Also note that at each stage, you add consecutive powers of 3; for example, stage 1 has $3^0 = 1$ white triangle; stage 2 has $3^0 + 3^1 = 4$ white triangles; stage 3 has $3^0 + 3^1 + 3^2 = 13$ white triangles; thus, stage 4 has $3^0 + 3^1 + 3^2 + 3^3 = 40$ white triangles.

Score: /3

Problem 2: Calculate the following:

a) $\begin{array}{r} 15 \\ \times\ 15 \\ \hline 225 \end{array}$ b) $\begin{array}{r} 25 \\ \times\ 25 \\ \hline 625 \end{array}$ c) $\begin{array}{r} 35 \\ \times\ 35 \\ \hline 1225 \end{array}$ d) $\begin{array}{r} 45 \\ \times\ 45 \\ \hline 2025 \end{array}$

Compare the answers with the numbers you multiplied. Look for a pattern to help you determine the following two product. 95×95 and 1000005×1000005

The last two digits must be 25, as most of you observed. The hundreds digit and larger are obtained by multiplying the common tens digit and the following integer. For example, $4 \times 5 = 20$, so $45 \times 45 = 2025$. More precisely, if you are given $(10a + 5) \times (10a + 5)$, then you get $100a(a + 1) + 25$.
Therefore, $95 \times 95 = 9025$ and $1000005 \times 1000005 = 1000010000025$.

Score: /3

Problem 3: Decide whether each equation is true.

a. $12 \times 42 = 21 \times 24$

b. $13 \times 62 = 31 \times 26$

c. $23 \times 96 = 32 \times 69$

Explain if this allows you to conclude that $37 \times 54 = 73 \times 45$?

No, $37 \times 54 = 1998$ while $73 \times 45 = 3285$, so this is a false pattern.
Indeed, if $(10a + b)(10c + d) = (10b + a)(10d + c)$, then
$100ac + 10ad + 10bc + bd = 100bd + 10bc + 10ab + ac$, so $99ac = 99bd$, so $ac = bd$, so the pattern holds only if the product of the tens-digits equals the product of the ones-digits.

Score: /2

Problem 4: As of September 2011, the three top-selling video games of all time, *Wii Sports*, *Super Mario Brothers*, and *Pokémon Red/Green/Blue*, sold a total of 118 million copies. If *Super Mario Brothers* sold 9 million more than *Pokémon* and 7 million less than *Wii Sports*, how many copies did *Pokémon* sell?

Let w be the number in million sold for *Wii Sports*, s for *Super Mario Brothers*, and p for *Pokémon Red/Green/Blue*, both in million.
$w + s + p = 118$ as given, and $s = 9 + p$, $s = w - 7$. Thus $p = s - 9$ and $w = s + 7$. This allows us to substitute the new expression into the first equation to obtain just an equation with s's. $s + 7 + s + s - 9 = 118$, so $3s = 120$ or $s = 40$. Thus $p = 40 - 9 = 31$.
So *Pokémon* sold 31 million copies.

Score: /2

Problem 5: In a farm with turkeys and sheep (at least one of each), suppose you count a total of 30 legs, how many of each kind may be on the farm?

Since each turkey has two legs and a sheep has four, we can tabulate the number of each below and check the number of legs in each case.

Turkey:	1	3	5	7	9	11	13
Sheep:	7	6	5	4	3	2	1

Suppose in addition, you count 24 eyes in total from the turkeys and sheep, how many of each kind do you have?

From the table, we see that a total of 12 animals giving 24 eyes only happened once, that is, 9 turkeys and 3 sheep.

Score: /3

Problem 6: First define both inductive reasoning and deductive reasoning.

a. Inductive reasoning:

The process of drawing a general conclusion by observing a pattern in specific instances is *inductive reasoning*. This can lead to a false conclusion.

b. Deductive reasoning:

One uses accepted facts, general principles, and logic to arrive at a specific conclusion.

Determine the type of reasoning the following situation illustrates: You use Google Maps' estimated time for a trip from Montreal to Québec City and calculate that to make that time, you will have to drive over 108 km/h where the speed limit on the highway is 100 km/h.

The conclusion is drawn by computing speed which is total distance from Montreal to Québec City divided by estimated time of travel. This is *deductive reasoning*.

Score: /3