Math 123
Fall 2017
Dr. Lily Yen

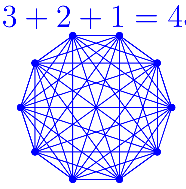
Test 1

Show all your work

Name: ____________________
Number: ____________________
Signature: ____________________
Score: ___/34

Problem 1: In the beginning of our class, a group of ten students shook hands with everyone else. How many handshakes would occur when they were all done?

Each of the 10 students shook hands with 9 others; total $10 \times 9 = 90$ handshakes. However, this way every handshake is counted twice, so the correct number is $90 \div 2 = 45$.
Alternatively, the first student shakes with 9 others. The second shakes with 8 others (not counting the first child). The third child shakes with 7 others, and so on. Total $9+8+7+6+5+4+3+2+1 = 45$ handshakes.

Alternatively, a picture:  each of the ten vertices represents a child, and each of the forty-five edges represents a handshake.

Score: /2

Problem 2: Suppose you want to share a long strip of fruit loop with your sisters, Annie and Betty. The strip is 60 cm long. Your strip is three times as long as Annie's; and Betty's strip is twice as long as Annie's. Find the length of each person's strip of fruit loop.

Say Annie's strip has length x. Then yours has length $3x$, and Betty's has length $2x$. Total $x + 3x + 2x = 6x$, but you know that the total is 60 cm, so $6x = 60$ cm, so $x = 10$ cm. Therefore, Annie's is $x = 10$ cm, Betty's is $2x = 20$ cm, and your is $3x = 30$ cm.

Score: /3

Problem 3: Solve this KenKen puzzle:

3− 1	2÷ 4	2	4+ 3
4	2 2	4+ 3	1
6× 2	3	1	2÷ 4
4+ 3	1	4 4	2

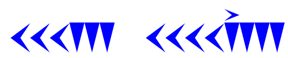
Score: /3

Problem 4: The movie *Gone with the Wind* was first shown in 1939. How would this year be specified in Roman numerals?

MCMXXXIX

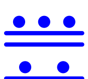
Score: /2

Problem 5: Express 2017 in Babylonian numeral.

Since $2017 = 33 \times 60 + 37$, the Babylonian numeral is 

Score: /2

Problem 6: Express 267 as a Mayan numeral.



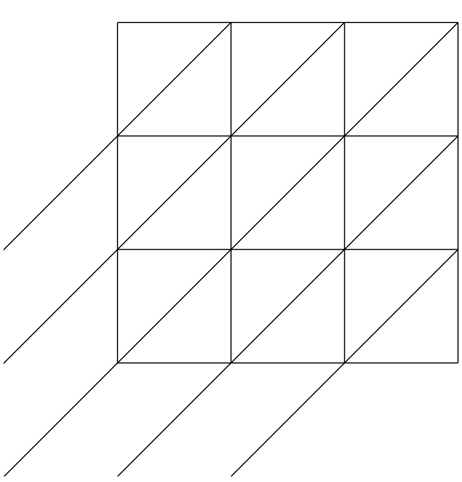
Score: /2

Problem 7: Convert 37 in base 10 to a binary numeral.

In base 10 we have that $37 = 32 + 4 + 1 = 2^5 + 2^2 + 2^0$. Therefore $37_{10} = 100101_2$.

Score: /2

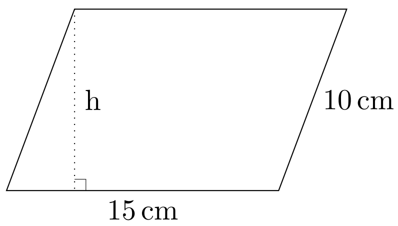
Problem 8: Multiply 931×254 using the galley method.



236 474

Score: /2

Problem 9: Given that the area of the parallelogram shown is $120\,\text{cm}^2$, find its height, h. Include units.



The area of a parallelogram is $A = bh = 15\,\text{cm} \times h = 120\,\text{cm}^2$.
So $h = 8\,\text{cm}$.
The length of the other side is just smoke and mirrors.

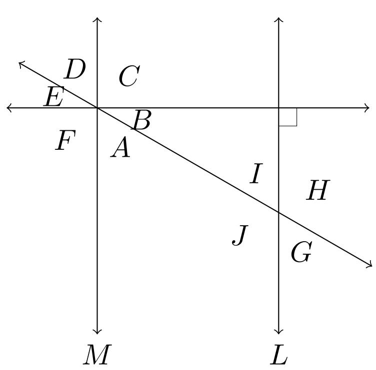
Score: /2

Problem 10: Lines L and M are parallel. Suppose Angle H is $120°$, find the following angles: **State the reason** for each answer.

a. Angle $J =$ $120°$

b. Angle $I =$ $60°$

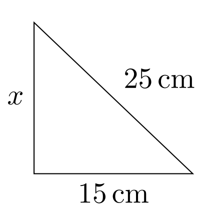
c. Angle $B =$ $30°$



Score: /3

Problem 11: Find the missing side length of the **right** triangle. If necessary, round to nearest thousandths.

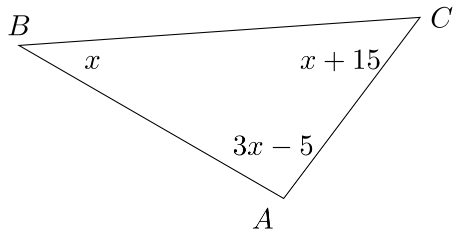
$20\,\text{cm}$



If the last side has length x, the Pythagorean Theorem gives that $x^2 + 21^2 = 29^2$, so $x^2 = 29^2 - 21^2 = 400$, so $x = 20$.

Score: /2

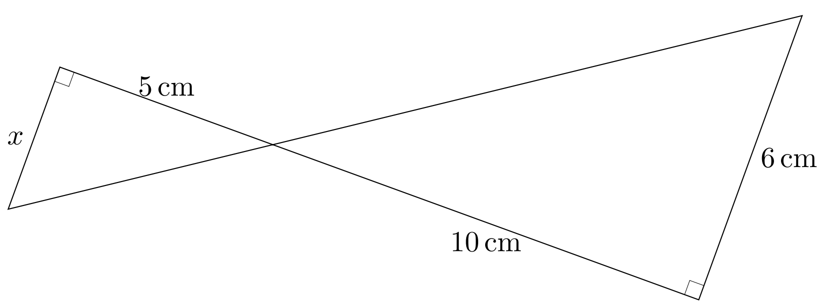
Problem 12: The measures of the angles of the triangle are indicated in terms of x. Find the measure of $\angle C =$ $49°$.



The angle sum is $x + (x + 15) + (3x - 5) = 5x + 10$, but the angle sum of any triangle is $180°$, so $5x + 10 = 180$, so $5x = 170$, so $x = 34$, so $\angle C = 34 + 15 = 49$.

Score: /3

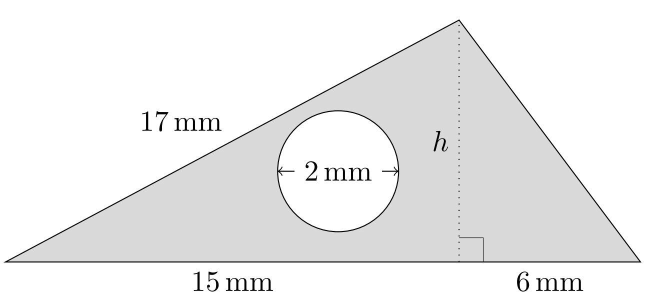
Problem 13: Find $x =$ $3\,\text{cm}$.



The two inner angles are vertically opposite so equal. Since each triangle has a right angle, the third angles have to be equal, too. Hence the triangles are similar and $\frac{x}{5} = \frac{6}{10}$, so $x = 5 \times \frac{6}{10} = 3$.

Score: /2

Problem 14: Find the **area** of the shaded region. Also find the **perimeter** of the big triangle.



Call the height of the largest triangle h. Since the left triangle is right-angled, $15^2 + h^2 = 17^2$, so $h^2 = 17^2 - 15^2 = 64$, so $h = 8$.
The base of the largest triangle is $15 + 6 = 21$, so the area of the triangle is $\frac{21\times 8}{2} = 84\,\text{mm}^2$.
The circle has radius $1\,\text{mm}$, so its area is $\pi\,\text{mm}^2$. Therefore, the area of the shaded region (in mm^2) is $84 - \pi \approx 80.86$.
To find the perimeter, we need the missing side via Pythagorean Theorem. Once we get $10\,\text{mm}$, then the perimeter is $17 + 10 + 15 + 6 = 48$ millimetres.

Score: /4