Math 116
Summer 2013
Dr. Lily Yen

# Test 3

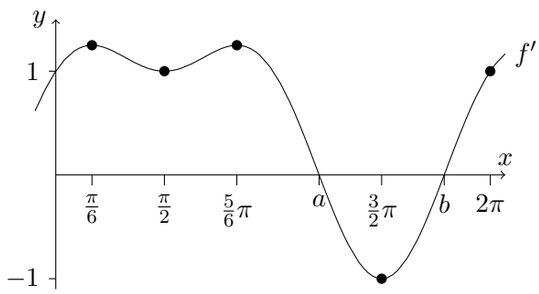
Show all your work

Name: _____

Score: ___/40

**No Calculator allowed in this part.**

**Problem 1**: The graph shows the derivative, $f'$, of $f$ on $[-\pi/6, 2\pi]$. Answer the following questions on the same interval.




a. Locate the $x$-coordinates of all points of inflection of $f$,

b. the local minima of $f$,

c. and the local maxima of $f$.

d. Determine the open intervals on which $f$ is increasing,

e. $f$ is decreasing, (give open interval(s))

f. $f$ is concave up, (give open interval(s))

g. and $f$ is concave down. (Give open interval(s).)

Score: /8

**Problem 2**: Differentiate the following without simplifying your answers.

a. $f(x) = \ln|\cos^{-1}(x)|$

Score: /2

b. Find $\dfrac{dy}{dx}$ for $\tan(x+y) = xy$ in terms of $x$ and $y$ only, without any derivative.

Score: /3

c. Find $g'(x)$ where $g(x) = (x+2)^{3+\sin(x)}$

Score: /4

**Problem 3**: Sketch the graph of a function $f$ satisfying all of the given conditions:

a. $f'(x) < 0$ for $x < 0$.

b. $f'(x) > 0$ for $x > 0$.

c. $f''(x) < 0$ for $|x| > 2$.

d. $f''(x) > 0$ for $|x| < 2$.

Score: /3

**Problem 4**: State the mean value theorem. Verify the theorem with $f(x) = \sqrt{x}$ on the interval $[1, 9]$.

Score: /5

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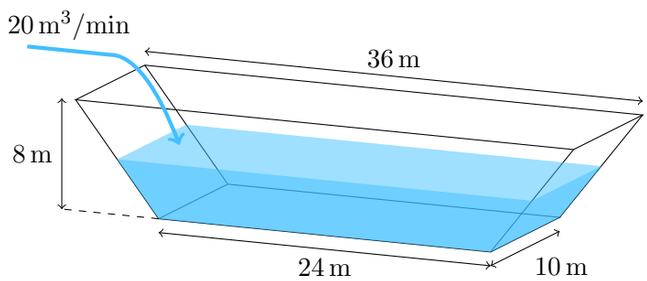
Calculators are allowed in the part.

**Problem 5**: How fast does the water level rise in the tank shown when the water level is $h = 4\,\text{m}$ and water pours in at 20 cubic metres per minute?




Score: /4

**Problem 6**: Compute the linearization, $L(x)$, of $f(x) = x^2 - x^{3/2}$ at $x = 4$. Find an interval around $x = 4$ such that the error is less than 0.1. Give 3 decimal places.

Score: /4

**Problem 7**: Sketch the curve $c(t) = (t^2 - 4t, 9 - t^2)$ for $t \in [-8, 8]$.

a. Mark clearly with V the point(s) where the curve has a vertical tangent and with H the point(s) where the curve has a horizontal tangent. Also label coordinates of the initial point, coordinates of the terminal point, and the orientation of $c(t)$. Score: /3

b. Find all point(s) $(x, y)$ where $c$ has vertical tangent(s).

Score: /2

c. Find the equation of a tangent line at $(-3, 8)$.

Score: /2