Math 108-01
Summer 2025
Dr. Lily Yen

Midterm II

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/30

One calculator (TI-83 or 84) is allowed for this test.

Problem 1: Find $g'(x)$ for $g(x) = \cos^2(5-x)\big((\tan^{-1}(\ln(x)) + e^{x^2-3x}\big)$.

Score: /3

Problem 2: Determine dy/dx for $x^3 + y^2 - x^2y = 7$.

a. For the implicitly defined function, find all points on the curve where $x = 1$. Find an equation of the tangent line for one of the points you found.

Score: /4

/7

Problem 3: The concentration (in µg/L) of a certain drug in the bloodstream x hours after being administered is approximately $C(x) = \dfrac{2x}{5+x^2}$. Use the differential to approximate the change in concentration from 2 hours after to 2.1 hours after the drug was administered. Provide 6 decimal places.

Score: /3

Problem 4: Use logarithmic differentiation to find y' for y.

$$y = \frac{(x+5)^2(3^x + \sin^{-1}(x))\log(x-2)}{x^2 - \sqrt{x+1}}$$

Score: /3

Problem 5: The total profit $P(x)$ (in thousands of dollars) from the sale of x hundred thousand automobile tires is approximated by

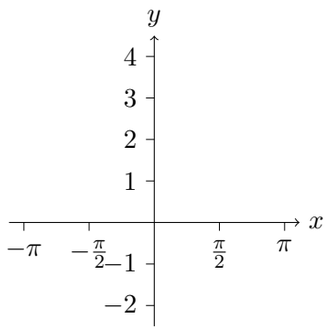
$$P(x) = -x^3 + 9x^2 + 120x - 400, \quad x \geq 5$$

Find the number of hundred thousands of tires that must be sold to maximize profit. Find the maximum profit.

Score: /3

/9

Problem 6: Graph $f(x) = x - \cos(x)$ in $[-\pi, \pi]$, and determine the exact x coordinates where f is increasing most rapidly and also where f is increasing least rapidly. Your work needs to include calculus techniques.



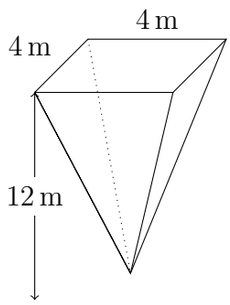
Score: /4

Problem 7: A tiny spherical balloon is inserted into a clogged artery and is inflated at a rate of 0.002π cubic millimetres per minute. How fast is the radius of the balloon increasing when the diameter is 0.010 mm?

Score: /3

/7

Problem 8: A tank shaped like an upside-down square ($4\,\text{m} \times 4\,\text{m}$) pyramid with height $12\,\text{m}$ is leaking water at the rate of $0.2\,\text{m}^3/\text{s}$. How fast does the height decrease when the water is $2\,\text{m}$ deep? Give 2 decimal places.



Score: /4

Problem 9: Graph a continuous function f satisfying all the following conditions over the domain $[-3, 3]$.

a. The function f is increasing for $(-3, -1) \cup (2, \infty)$.

b. The first derivative of f is negative, namely, $f'(x) < 0$ for $(-1, 2)$.

c. The function is concave down for $x \in (-3, 2) \cup (2, 3)$.

Score: /3

/7