

Math 108-01
Summer 2025
Dr. Lily Yen

Midterm One

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ____/40

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞ , and $-\infty$ where appropriate.

a. $\lim_{x \rightarrow -\infty} \frac{x^3 - 2 - 5x}{3x^3 + (x - 4)^2} =$

Score: /2

b. $\lim_{x \rightarrow 2} \frac{\sqrt{x + 2} - 2}{x - 2} =$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find the derivative of $h(x) = 3x^3 - 6\sqrt{x} + 1\pi$.

Score: /2

b. Find $f'(x)$ where $f(x) = (2x^3 - \sin(x))(e^x - \sqrt[3]{x} + \pi)$

Score: /2

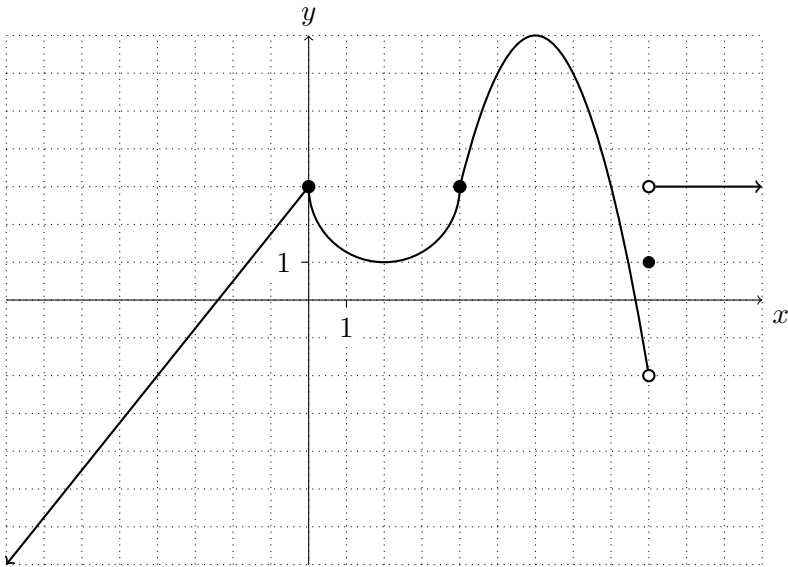
c. Find $d(g(x))/dx$ where

$$g(x) = \frac{\tan(x) - x\sqrt{x}}{2 - 5x^4 - \frac{1}{x}}$$

Score: /2

Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown, piece-wise defined by lines, a parabola, and a semi-circle. Use the graph to answer the questions. Use the symbols DNE, ∞ , and $-\infty$ where appropriate.



a. In the domain of the function f on $(-\infty, \infty)$, list all x where f is discontinuous.

b. List the x values where f is continuous but not differentiable.

c. $\lim_{x \rightarrow -9^+} f(x) =$

d. $\lim_{x \rightarrow -9^-} f(x) =$

e. $\lim_{x \rightarrow 6} f'(x) =$

f. $\lim_{h \rightarrow 0^-} \frac{f(4+h) - f(4)}{h} =$

g. $\lim_{x \rightarrow -\pi} \frac{f(x) - f(-\pi)}{x + \pi} =$

h. Estimate $f'(5)$ by drawing a tangent line at the point in question and approximating its slope.

i. In the same grid above, graph $y = f'(x)$ for the entire x -axis.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = -1$?

$$g(x) = \begin{cases} 2x - x^3 + c, & x < -1 \\ 3x + 5, & x \geq -1 \end{cases}$$

Score: /4

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \frac{1}{1-x}$.

b. Find an equation of the tangent line to f at $x = 3$.

Score: /5

Problem 6: Suppose the profit function (in dollars) of a new Jazz Club at Capilano University t months after opening is given by

$$P(t) = \frac{1000}{1 + e^{-2t}}$$

Use a graphing calculator (TI83, TI83+, TI84-Plus) to set up a table of values to estimate the instantaneous rate of profit half a year after opening. Round your answers to 6 decimal places. Specify your Y_1 and Y_2 as part of your steps.

Interval	$Y_2 =$
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Score: /3

Problem 7: The spread of an avian flu virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds) with the virus, and t is the number of weeks since the first case was observed at Capilano University’s main campus. Carefully interpret the following mathematical statements regarding the virus.

a. $\frac{\Delta V}{\Delta t} = 0.5$ for $t = 1$ and $t = 4$.

b. $V'(17) = -0.2$

Score: /2

Problem 8: Shown is a sample of 10 classrooms at CapU during fire alarm drill week showing the class size and the number of minutes it took each class to vacate the room once the alarm started ringing.

Classroom size (students):	27	36	32	41	29	21	39	43	86	78
Time to vacate (in minutes):	2.6	3.2	2.8	3.4	2.1	2.4	3.3	3.6	4.2	3.9

Use the given data to answer the following questions:

- a. Draw a scatter plot. Provide dimensions of the window and label your axes according to the sample data.

Score: /2

- b. Use linear regression to find a model to fit your plot. Report your model to six decimal places.

Score: /2

- c. According to your model, what is the time accurate to a tenth of a minute of a classroom with 35 students would take to vacate when the fire alarm starts to ring? Comment on the reliability of your answer.

Score: /1

- d. Use your model to estimate class size if a class requires 4 minutes to vacate the room.

Score: /1