Math 108-01
Summer 2025
Dr. Lily Yen

Midterm 1

Show all your work

Name: ____________
Number: ____________
Signature: ____________
Score: ___/40

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\lim_{x\to-\infty} \frac{x^3-5-4x}{4x^3+(x-2)^2} =$ $\boxed{\frac{1}{4}}$

$$\lim_{x\to-\infty} \frac{x^3-5-4x}{4x^3+(x-2)^2} = \lim_{x\to-\infty} \frac{x^3\left(1-\frac{5}{x^3}-\frac{4}{x^2}\right)}{\left(4+\frac{(x-2)^2}{x^3}\right)x^3} = \lim_{x\to-\infty} \frac{1-\frac{5}{x^3}-\frac{4}{x^2}}{4+\frac{(x-2)^2}{x^3}} = \frac{1}{4}$$

Score: /2

b. $\lim_{x\to3} \frac{\sqrt{x+1}-2}{x-3} =$ $\boxed{\frac{1}{4}}$

$$\lim_{x\to3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to3} \frac{\left(\sqrt{x+1}-2\right)\left(\sqrt{x+1}+2\right)}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{\sqrt{x+1}^2-2^2}{(x-3)\left(\sqrt{x+1}+2\right)}$$

$$= \lim_{x\to3} \frac{x+1-4}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{x-3}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x\to3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find the derivative of $h(x) = 2x^3 - 4\sqrt{x} + 5\pi$.

$$h'(x) = 6x^2 - \frac{4}{2\sqrt{x}} + 0 = 6x^2 - \frac{2}{\sqrt{x}}$$

Score: /2

b. Find $f'(x)$ where $f(x) = (5x^3 - \sin(x))(e^x - \sqrt[4]{x} + \pi)$

$$f'(x) = \left(15x^2 - \cos(x)\right)\left(e^x - \sqrt[4]{x} + \pi\right)\left(5x^3 - \sin(x)\right)\left(e^x - \tfrac{1}{4}x^{-3/4}\right)$$

Score: /2

c. Find $d(g(x))/dx$ where

$$g(x) = \frac{\tan(x) - x\sqrt{x}}{6 - 3x^4 - \frac{1}{x}}$$

$$d(g(x))/dx = \frac{\left(\sec^2(x) - \frac{3}{2}\sqrt{x}\right)\left(6 - 3x^4 - \frac{1}{x}\right) - \left(\tan(x) - x\sqrt{x}\right)\left(-12x^3 + \frac{1}{x^2}\right)}{\left(6 - 3x^4 - \frac{1}{x}\right)^2}$$

Score: /2

/10

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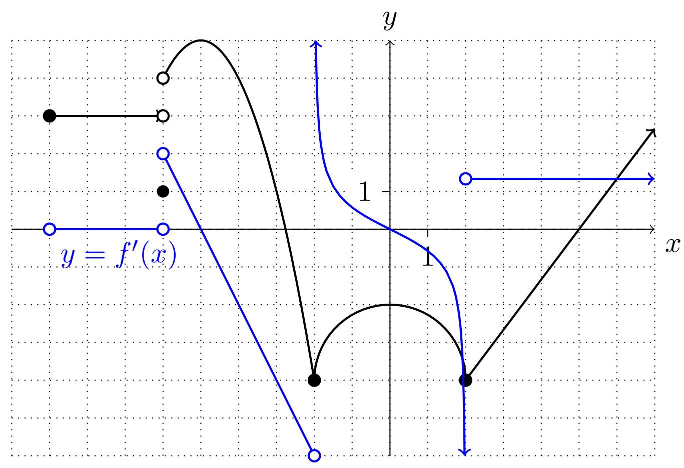
Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown, piece-wise defined by lines, a parabola, and a semi-circle. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. In the domain of the function f on $[-9, \infty)$, list all x where f is discontinuous.

$x = -6$

b. List the x values where f is continuous but not differentiable.

$x = -2, 2$

c. $\lim_{x \to -6^+} f(x) =$ **4**

d. $\lim_{x \to -6^-} f(x) =$ **3**

e. $\lim_{x \to -5} f'(x) =$ **0**

f. $\lim_{h \to 0^+} \frac{f(-2+h) - f(-2)}{h} =$ **∞**

g. $\lim_{x \to \pi} \frac{f(x) - f(\pi)}{x - \pi} =$ **4/3**

h. Estimate $f'(-3)$ by drawing a tangent line at the point in question and approximating its slope.

-4

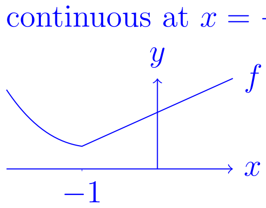
i. In the same grid above, graph $y = f'(x)$ for the interval $[-9, 7]$.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = -1$?

$$g(x) = \begin{cases} 2x - x^3 + c, & x < -1 \\ 3x + 5, & x \geq -1 \end{cases}$$

Polynomials are continuous, so each piece of g is continuous.
Note that $\lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} 2x - x^3 + c = -1 + c$, that $\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} 3x + 5 = 2$, and that $f(-1) = 3(-1) + 5 = 2$. Therefore f is continuous at $x = -1$ (and hence everywhere) if $-1 + c = 2$, so $c = 3$.



The graph looks like it has a cusp at $(-1, 2)$, so f is likely NOT DIFFERENTIABLE and further analysis bears this out.

Score: /4

/14

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{1-x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{1-x-h} - \frac{1}{1-x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1-x}{(1-x-h)(1-x)} - \frac{1-x-h}{(1-x-h)(1-x)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{h}{(1-x-h)(1-x)}}{h} = \lim_{h\to 0} \frac{1}{(1-x-h)(1-x)}$$

$$= \frac{1}{(1-x)^2}$$

b. Find an equation of the tangent line to f at $x = 2$.

Since $f(2) = -1$ and $f'(2) = 1$, the tangent line is

$$y + 1 = 1(x-2) \quad \text{or} \quad y = x - 3$$

Score: /5

Problem 6: Suppose the profit function (in dollars) of a new Jazz Club at Capilano University t months after opening is given by

$$P(t) = \frac{1000}{1 + e^{-2t}}$$

Use a graphing calculator (TI83, TI83+, TI84-Plus) to set up a table of values to estimate the instantaneous rate of profit half a year after opening. Round your answers to 6 decimal places. Specify your Y_1 and Y_2 as part of your steps.

Interval	$Y_2 = (Y_1(X) - 999.99)/(X - 6)$
5.8000 to 6.0000	0.015 109
5.9000 to 6.0000	0.013 603
5.9990 to 6.0000	0.012 301
6.1000 to 6.0000	0.011 137
6.0100 to 6.0000	0.012 166
6.0010 to 6.0000	0.012 276

$P'(6) \approx \$0.012\,288$/month.

Score: /3

/8

Problem 7: The spread of an avian flu virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds) with the virus, and t is the number of weeks since the first case was observed at Capilano University's main campus. Carefully interpret the following mathematical statements regarding the virus.

a. $\dfrac{\Delta V}{\Delta t} = 0.4$ for $t = 1$ and $t = 3$.

In the second through fourth week the number of infected people grew at an average rate of 40 people per week.

b. $V'(16) = -0.1$

Seventeen weeks after the first case, the number of infected people is dropping at a rate of 10 people per week.

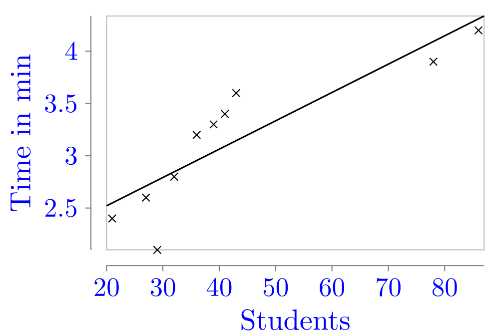
Score: /2

Problem 8: Shown is a sample of 10 classrooms at CapU during fire alarm drill week showing the class size and the number of minutes it took each class to vacate the room once the alarm started ringing.

Classroom size (students):	27	36	32	41	29	21	39	43	86	78
Time to vacate (in minutes):	2.6	3.2	2.8	3.4	2.1	2.4	3.3	3.6	4.2	3.9

Use the given data to answer the following questions:

a. Draw a scatter plot. Provide dimensions of the window and label your axes according to the sample data.



Score: /2

b. Use linear regression to find a model to fit your plot. Report your model to six decimal places.

$$y = 0.027\,098x + 1.979\,374$$

Score: /2

c. According to your model, what is the time accurate to a tenth of a minute of a classroom with 35 students would take to vacate when the fire alarm starts to ring? Comment on the reliability of your answer.

If $x = 35$, then $y \approx 2.9$ min.
Interpolation is valid, so reliable.

Score: /1

d. Use your model to estimate class size if a class requires 4 minutes to vacate the room.

Solving $4 = 0.027\,098x + 1.979\,374$ yields that $x \approx 74.6$. Thus around 75 students.

Score: /1

/8