Math 108-01
Summer 2025
Dr. Lily Yen

Quiz 5

Show all your work

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Score: ___/10

Problem 1: Use linear approximation to estimate $\sqrt[4]{15.99}$ to 4 decimal places.

If $f(x) = \sqrt[4]{x}$, and $a = 16$, then $f'(x) = \frac{1}{4}x^{-3/4}$, $f(a) = \sqrt[4]{16} = 2$, and $f'(a) = \frac{1}{4\times(16)^{(3/4)}} = \frac{1}{4\times 2^3} = \frac{1}{32}$, so

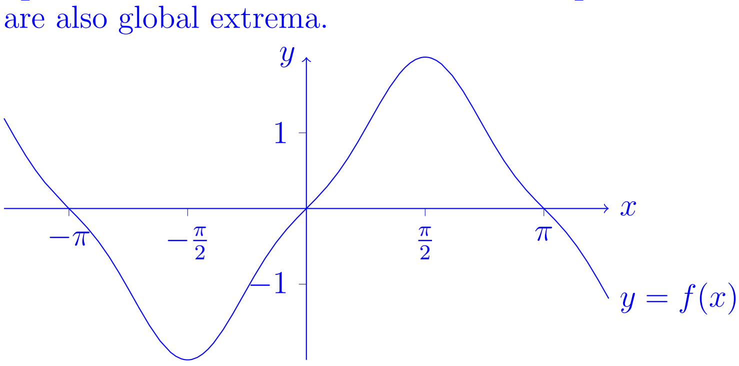
$$\sqrt[4]{15.99} = f(15.99) \approx f(a) + f'(a)(15.99 - a) = 2 + \tfrac{1}{32}(-0.01) \approx 1.9997$$

Score: /3

Problem 2: Determine the intervals where the given function $f(x) = \sin(x) + \sin^3(x)$ is increasing; and find all local and global extrema (both coordinates) in the interval $[-\pi, \pi]$. Give 4 decimal places for approximations. Hint: Drawing f is helpful.

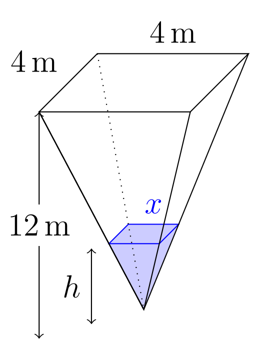
$$f'(x) = \cos(x) + 3\sin^2(x)\cos(x) = \big((1 + 3\sin^2(x)\big)\cos(x),$$

and since $1 + 3\sin^2(x) \geq 1$ (so never zero), $f'(x) = 0$ when $\cos(x) = 0$, so at $\frac{\pi}{2} + n\pi$, $n \in \mathbb{Z}$. Moreover, the sign of $f'(x)$ is the same as the sign of $\cos(x)$. Checking the sign of f' around the critical points, one determines whether the point is a local max or min.
Now $f(\frac{\pi}{2} + 2n\pi) = \sin(\frac{\pi}{2}) + \sin^3(\frac{\pi}{2}) = 1 + 1^3 = 2$, and
$f(-\frac{\pi}{2} + 2n\pi) = \sin(-\frac{\pi}{2}) + \sin^3(-\frac{\pi}{2}) = -1 + (-1)^3 = -2$, so the local maxima are $(\frac{\pi}{2} + 2n\pi, 2)$ and the local minima are $(-\frac{\pi}{2} + 2n\pi, -2)$. Within the domain, $[-\pi, \pi]$, they are also global extrema.



Score: /3

Problem 3: A tank shaped like an upside-down square (4 m × 4 m) pyramid with height 12 m is leaking water at the rate of $0.2\,\mathrm{m}^3/\mathrm{s}$. How fast does the height decrease when the water is 2 m deep? Give 4 decimal places.



Let x be the side length of the square at the top of the water, and let h be the depth of the water. By similar triangles, $\frac{x}{4} = \frac{h}{12}$, so $x = h/6$. The volume of the water is $V = \frac{1}{3}x^2h = \frac{1}{3}(h/3)^2h = h^3/27$. Therefore

$$\frac{dV}{dt} = \frac{h^2}{9}\frac{dh}{dt}.$$

When $h = 2\,\mathrm{m}$,

$$\frac{dh}{dt} = \frac{dV/dt}{h^2/9} = \frac{-0.2\,\mathrm{m}^3/\mathrm{s}}{(2\,\mathrm{m})^2/9} = -0.45\,\mathrm{m/s}.$$

Score: /4