Math 108-01
Summer 2025
Dr. Lily Yen

Quiz 4

Show all your work

Name: ____________

Number: ____________

Signature: ____________

Score: ___/10

Problem 1: Answer the following using derivative rules. Do NOT simplify.

a. Find $g'(x)$ where $g(x) = \sin^2(x+3)\left(e^{x^3-x} + \tan^{-1}(\ln(x))\right)$

$$g'(x) = 2\sin(x+3)\cos(x+3)\left(e^{x^3-x} + \tan^{-1}(\ln(x))\right) + \sin^2(x+3)\left((3x^2-1)e^{x^3-x} + \frac{1}{(\ln^2(x)+1)x}\right)$$

Score: /3

b. Find $d(f(x))/dx$ where

$$f(x) = \frac{5^x + \sec(x+\sqrt{x})}{\log(x^3+\cos(x))}$$

$$f'(x) = \frac{\left(5^x\ln(5) + \sec(x+\sqrt{x})\tan(x+\sqrt{x})\left(1+\frac{1}{2\sqrt{x}}\right)\right)\log(x^3+\cos(x)) - \left(5^x + \sec(x+\sqrt{x})\right)\frac{3x^2-\sin(x)}{(x^3+\cos(x))\ln 10}}{\left(\log(x^3+\cos(x))\right)^2}$$

Score: /3

Problem 2: Use implicit differentiation to determine dy/dx for $y^2 + 2y + 2x^2 - 2xy = 27$. Also, find an equation of the tangent line at a point on the curve where $x = 1$. One equation is enough.

If $y^2 + 2y + 2x^2 - 2xy = 27$, then $2yy' + 2y' + 4x - 2y - 2xy' = 0$, so $2yy' + 2y' - 2xy' = 2y - 4x$, so $(2y + 2 - 2x)y' = 2y - 4x$, so

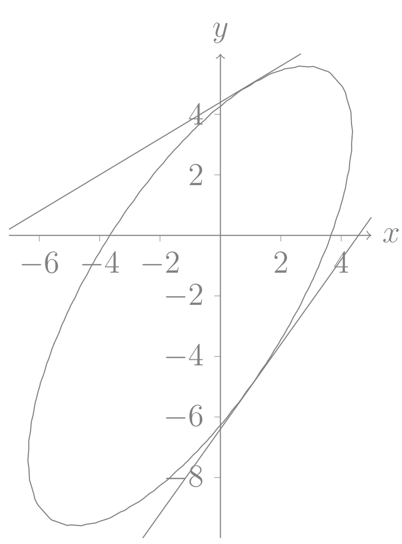
$$y' = \frac{2y-4x}{2y+2-2x} = \frac{y-2x}{y+1-x}$$

If $x = 1$ in $y^2 + 2y + 2x^2 - 2xy = 27$, then $y^2 + 2 = 27$, so $y^2 = 25$, so $y = \pm 5$. Now,

$$y'\Big|_{(1,5)} = \frac{5-2}{5+1-1} = \frac{3}{5} \qquad \text{while} \qquad y'\Big|_{(1,-5)} = \frac{-5-2}{-5+1-1} = \frac{-7}{-5} = \frac{7}{5},$$

so the two tangent lines are

$$y - 5 = \frac{3}{5}(x-1) \quad \text{and} \quad y + 5 = \frac{7}{5}(x-1)$$



Score: /4