Math 108-01
Summer 2024
Dr. Lily Yen

Midterm One

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/45

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\lim_{x\to 6^+} \dfrac{x^2-5x-6}{|6-x|} =$

Score: /2

b. $\lim_{x\to 3} \dfrac{x-7}{(x-3)^2} =$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find $h'(x)$ where $h(x) = \sin\left(\dfrac{1}{x} - \log_3(x^2)\right)$

Score: /3

b. Find $d(g(x))/dx$ where

$$g(x) = \frac{\tan^{-1}(e^{6x})}{(\pi + 2x^3 - 5\sqrt[4]{x})}$$

Score: /3

/10

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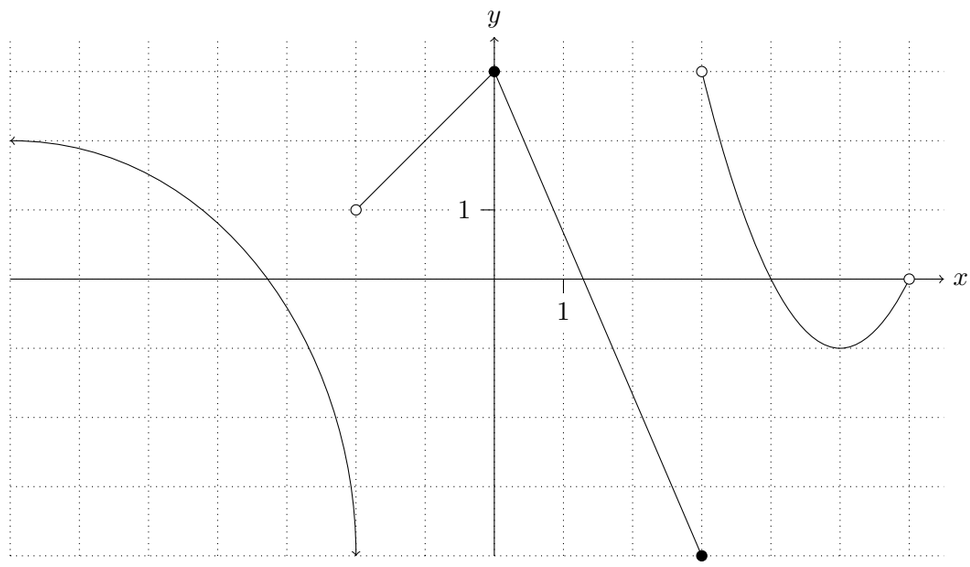
Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. Express in as few intervals as possible where f is continuous in $(-\infty, 6)$.

b. List the x values where f is continuous but not differentiable.

c. $\lim_{x \to 2^-} f(x) =$

d. $\lim_{x \to 3^+} f(x) =$

e. $\lim_{x \to -\infty} f(x) =$

f. $\lim_{x \to 0} \dfrac{f(x) - f(0)}{x} =$

g. $\lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h} =$

h. Estimate $f'(-3)$ by drawing a tangent line at the point in question and approximating its slope.

i. In the same grid above, graph $y = f'(x)$ for the interval $(-2, 6)$ where you see a piece-wise linear function and a parabola.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = -2$?

$$f(x) = \begin{cases} x^3 - 1, & x \leq -2 \\ 3x + c, & x > -2 \end{cases}$$

Score: /4

/14

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{x+3}$.

b. Find an equation of the tangent line to f at $x = 2$.

Score: /5

Problem 6: Given the following implicitly defined function:

$$y^2 + 2x^2 + 2y - 2xy = 30$$

a. Solve for $\dfrac{dy}{dx}$.

b. Find all point(s) on the curve with a tangent slope of 2.

Score: /5

/10

Problem 7: The spread of an avian flu virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds) with the virus, and t is the number of weeks since the first case was observed at Capilano University's main campus. Carefully interpret the following mathematical statements regarding the virus.

a. $V'(3) = 0.4$

b. $\dfrac{\Delta V}{\Delta t} = 0.3$ for $t = 0$ and $t = 5$.

Score: /2

Problem 8: Capilano University East Indian Truck food company has found the following cost/production information:

Lunch boxes produced:	0	5	15	35	45	50	60
Total cost ($) of production:	230	270	325	450	500	535	590

a. Sketch the scatterplot and find the linear model.

Score: /4

b. Find the marginal cost function from the model.

Score: /2

c. With your model, approximate the cost of producing the 16th lunch box.

Score: /1

d. Find the average cost of producing x lunch boxes.

Score: /2

/11