Math 108-01
Summer 2024
Dr. Lily Yen

Midterm 1

Show all your work

Name: ____________

Number: ____________

Signature: ____________

Score: ___/45

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\displaystyle\lim_{x\to 2}\frac{x-5}{(x-2)^2} =$ $\boxed{-\infty}$

Since $\lim_{x\to 2} x-5 = -3$ and $\lim_{x\to 2}(x-2)^2 = 0$ (while staying positive),

$$\lim_{x\to 2}\frac{x-5}{(x-2)^2} = -\infty$$

Score: /2

b. $\displaystyle\lim_{x\to 7^+}\frac{x^2-6x-7}{|7-x|} =$ $\boxed{8}$

If $x > 7$, then $|7-x| = -(7-x) = x-7$, so

$$\lim_{x\to 7^+}\frac{x^2-6x-7}{|7-x|} = \lim_{x\to 7^+}\frac{(x-7)(x+1)}{x-7} = \lim_{x\to 7^+} x+1 = 8$$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find $h'(x)$ where $h(x) = \log_3\left(\frac{1}{x} - \sin(x^2)\right)$

$$h'(x) = \frac{1}{\left(\frac{1}{x} - \sin(x^2)\right)\ln(3)}\left(\frac{-1}{x^2} - \cos(x^2)2x\right)$$

Score: /3

b. Find $d(g(x))/dx$ where

$$g(x) = \frac{\cos^{-1}(e^{2x})}{(\pi + 3x^4 - 7\sqrt[3]{x})}$$

$$g'(x) = \frac{\frac{-1}{\sqrt{1-e^{4x}}}e^{2x}2\left(\pi + 3x^4 - 7\sqrt[3]{x}\right) - \cos^{-1}(e^{2x})\left(12x^3 - \frac{7}{3}x^{-2/3}\right)}{\left(\pi + 3x^4 - 7\sqrt[3]{x}\right)^2}$$

Score: /3

/10

Math 108-01
Summer 2024
Dr. Lily Yen

Midterm 1

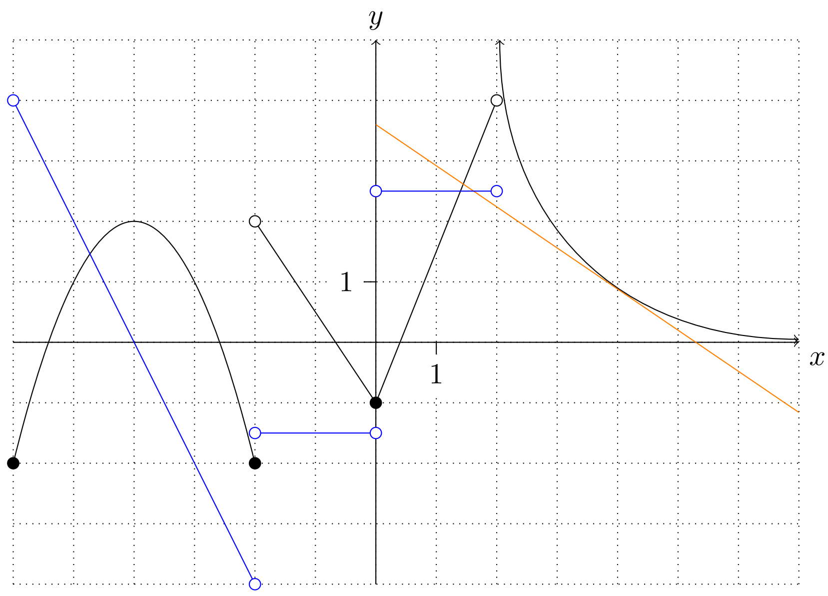
Show all your work

Name: ____________________

Number: ____________________

Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. Express in as few intervals as possible where f is continuous in $[-6, \infty)$.

$[-6, -2) \cup (-2, 2) \cup (2, \infty)$

b. List the x values where f is continuous but not differentiable.

$x = 0$

c. $\lim_{x \to 2^+} f(x) =$ ∞

d. $\lim_{x \to \infty} f(x) =$ 0

e. $\lim_{x \to -2^-} f(x) =$ -2

f. $\lim_{h \to 0} \dfrac{f(h) - f(0)}{h} =$ DNE

g. $\lim_{x \to -1} \dfrac{f(x) - f(-1)}{x + 1} =$ $-3/2$

h. Estimate $f'(4)$ by drawing a tangent line at the point in question and approximating its slope.

-0.68

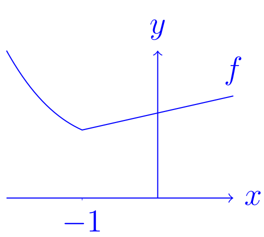
i. In the same grid above, graph $y = f'(x)$ for the interval $[-6, 2)$ where you see a parabola and a piece-wise linear function.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = -1$?

$$g(x) = \begin{cases} 5 - x^3, & x < -1 \\ 2x + c, & x \geq -1 \end{cases}$$

Polynomials and roots are continuous. Compositions of continuous functions are continuous. Therefore each piece of g is continuous.
Note that $\lim_{x \to -1^-} g(x) = \lim_{x \to -1^-} 5 - x^3 = 6$, that $\lim_{x \to -1^+} g(x) = \lim_{x \to -1^+} 2x + c = -2 + c$, and that $f(-1) = -2 + c$. Therefore f is continuous at $x = -1$ (and hence everywhere) if $-2 + c = 6$, so $c = 8$.



The graph looks like it has a cusp at $(-1, 6)$, so f is likely NOT DIFFERENTIABLE and further analysis bears this out.

Score: /4

/14

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{x+2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{x+2}{(x+h+2)(x+2)} - \frac{x+h+2}{(x+h+2)(x+2)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-h}{(x+h+2)(x+2)}}{h} = \lim_{h\to 0} \frac{-1}{(x+h+2)(x+2)}$$
$$= \frac{-1}{(x+2)^2}$$

b. Find an equation of the tangent line to f at $x = 3$.

Since $f(3) = \frac{1}{5}$ and $f'(3) = \frac{-1}{5^2} = -\frac{1}{25}$, the tangent line is

$$y - \frac{1}{5} = -\frac{1}{25}(x-3) \quad \text{or} \quad y = -\frac{1}{25}x + \frac{8}{25} \quad \text{or} \quad x + 25y - 8 = 0$$

Score: /5

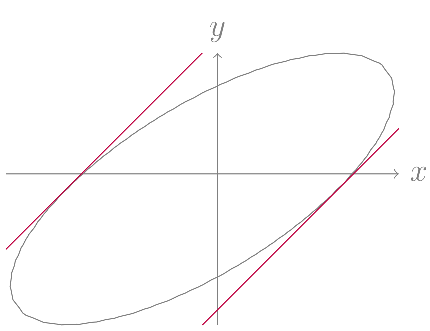
Problem 6: Given the following implicitly defined function:

$$x^2 + 2y^2 + 2y - 2xy = 80$$

a. Solve for $\dfrac{dy}{dx}$.

Here $2x + 4yy' + 2y' - 2(y + xy') = 0$, so $2x + 4yy' + 2y' - 2y - 2xy' = 0$, so $4yy' + 2y' - 2xy' = 2y - 2x$, so $(4y + 2 - 2x)y' = 2y - 2x$, so

$$\frac{dy}{dx} = \frac{2y-2x}{4y+2-2x} = \frac{y-x}{2y+1-x}$$



b. Find all point(s) on the curve with a tangent slope of 1.

If $\frac{y-x}{2y+1-x} = 1$, then $y - x = 2y + 1 - x$, so $y = -1$ is the only solution.
If $y = -1$ and $x^2 + 2y^2 + 2y - 2xy = 80$, then $x^2 + 2x = 80$, so $x^2 + 2x - 80 = 0$, so $(x-8)(x+10) = 0$, so $x = 8$ or $x = -10$.
Therefore the desired points of tangency are $(8, -1)$ and $(-10, -1)$.

Score: /5

/10

Problem 7: The spread of an avian flu virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds) with the virus, and t is the number of weeks since the first case was observed at Capilano University's main campus. Carefully interpret the following mathematical statements regarding the virus.

a. $\dfrac{\Delta V}{\Delta t} = 0.2$ for $t = 0$ and $t = 4$.

Four weeks after the first case, the number of infected people has grown at an average rate of 20 people per week.

b. $V'(4) = 0.5$

Four weeks after the first case, the number of infected people is growing at a rate of 50 people per week.

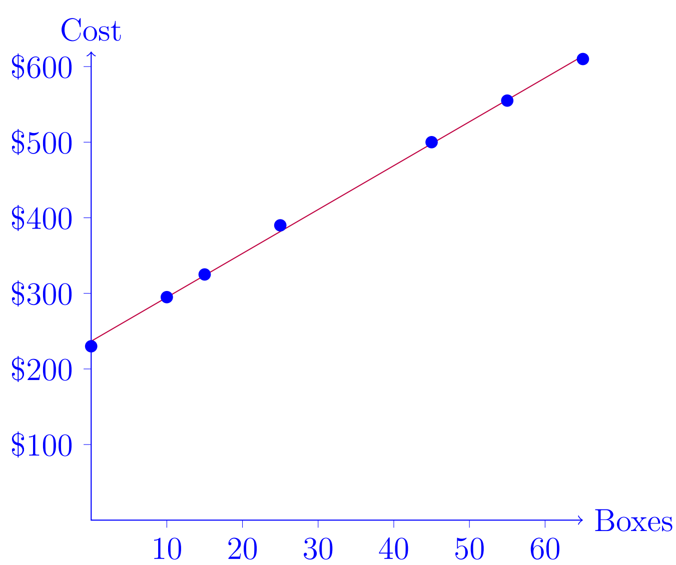
The two statements do not contradict each other since one would expect the rate to be small in the beginning.

Score: /2

Problem 8: Capilano University East Indian Truck food company has found the following cost/production information:

Lunch boxes produced:	0	10	15	25	45	55	65
Total cost ($) of production:	230	295	325	390	500	555	610

a. Sketch the scatterplot and find the linear model.



$y = 5.806x + 236.7$

Score: /4

b. Find the marginal cost function from the model.

$y' =$ $5.806 per box.

Score: /2

c. With your model, approximate the cost of producing the 14th lunch box.

$5.81 for producing the 14th lunch box.

Score: /1

d. Find the average cost of producing x lunch boxes.

$x \cdot \Delta y / \Delta x = 5.806x$ dollars.

Score: /2

/11