Math 108-01
Summer 2024
Dr. Lily Yen

Midterm 1

Show all your work

Name: _____
Number: _____
Signature: _____
Score: ___/45

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\lim_{x\to 2} \dfrac{x-5}{(x-2)^2} =$

Score: /2

b. $\lim_{x\to 7^+} \dfrac{x^2-6x-7}{|7-x|} =$

Score: /2

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find $h'(x)$ where $h(x) = \log_3\left(\dfrac{1}{x} - \sin(x^2)\right)$

Score: /3

b. Find $d(g(x))/dx$ where

$$g(x) = \frac{\cos^{-1}(e^{2x})}{(\pi + 3x^4 - 7\sqrt[3]{x})}$$

Score: /3

/10

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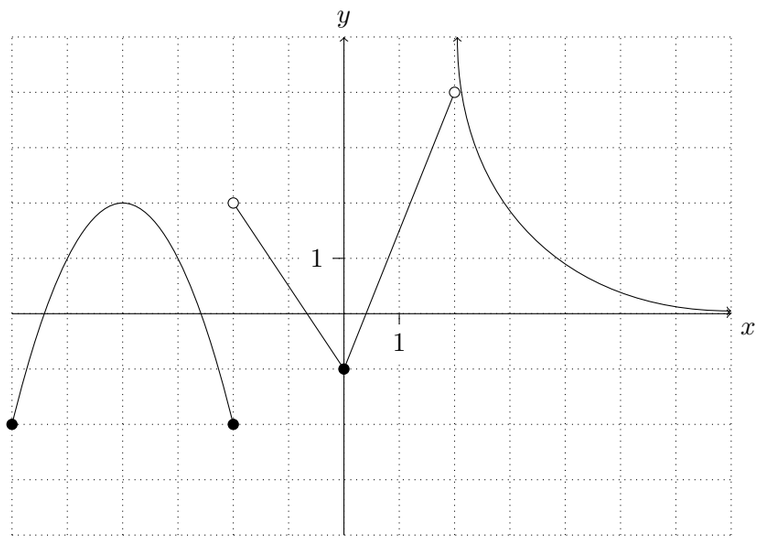
Calculators allowed from here on.

Problem 3: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. Express in as few intervals as possible where f is continuous in $[-6, \infty)$.

b. List the x values where f is continuous but not differentiable.

c. $\lim_{x \to 2^+} f(x) =$

d. $\lim_{x \to \infty} f(x) =$

e. $\lim_{x \to -2^-} f(x) =$

f. $\lim_{h \to 0} \dfrac{f(h) - f(0)}{h} =$

g. $\lim_{x \to -1} \dfrac{f(x) - f(-1)}{x + 1} =$

h. Estimate $f'(4)$ by drawing a tangent line at the point in question and approximating its slope.

i. In the same grid above, graph $y = f'(x)$ for the interval $[-6, 2)$ where you see a parabola and a piece-wise linear function.

Score: /10

Problem 4: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = -1$?

$$g(x) = \begin{cases} 5 - x^3, & x < -1 \\ 2x + c, & x \geq -1 \end{cases}$$

Score: /4

/14

Problem 5: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \dfrac{1}{x+2}$.

b. Find an equation of the tangent line to f at $x = 3$.

Score: /5

Problem 6: Given the following implicitly defined function:

$$x^2 + 2y^2 + 2y - 2xy = 80$$

a. Solve for $\dfrac{dy}{dx}$.

b. Find all point(s) on the curve with a tangent slope of 1.

Score: /5

/10

Problem 7: The spread of an avian flu virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds) with the virus, and t is the number of weeks since the first case was observed at Capilano University's main campus. Carefully interpret the following mathematical statements regarding the virus.

a. $\dfrac{\Delta V}{\Delta t} = 0.2$ for $t = 0$ and $t = 4$.

b. $V'(4) = 0.5$

Score: /2

Problem 8: Capilano University East Indian Truck food company has found the following cost/production information:

Lunch boxes produced:	0	10	15	25	45	55	65
Total cost ($) of production:	230	295	325	390	500	555	610

a. Sketch the scatterplot and find the linear model.

Score: /4

b. Find the marginal cost function from the model.

Score: /2

c. With your model, approximate the cost of producing the 14th lunch box.

Score: /1

d. Find the average cost of producing x lunch boxes.

Score: /2

/11