Math 108-01
Summer 2024
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Quiz Five

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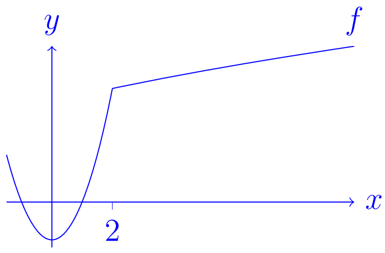
Score: ___/10

Problem 1: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} x^2 - 1, & x \le 2 \\ \sqrt{x - c}, & x > 2 \end{cases}$$

Polynomials and roots are continuous. Compositions of continuous functions are continuous. Therefore each piece of f is continuous.
Note that $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 - 1 = 3$, that $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \sqrt{x-c} = \sqrt{2-c}$, and that $f(2) = 3$. Therefore f is continuous at $x = 2$ (and hence everywhere) if $\sqrt{2-c} = 3$, so $2 - c = 9$, so $c = -7$.



The graph looks like it has a cusp at $(2, 3)$, so f is likely NOT DIFFERENTIABLE and further analysis bears this out.

Score: /4

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find $g'(x)$ where $g(x) = \left(4x^3 + \frac{1}{x^3} - 50\right)(x^2 - 2\sqrt{x} + e)$

$$g'(x) = \left(12x^2 - \frac{3}{x^4}\right)(x^2 - 2\sqrt{x} + e) + \left(4x^3 + \frac{1}{x^3} - 50\right)\left(2x - \frac{1}{\sqrt{x}}\right)$$
$$= 20x^4 - 28x^{5/2} + 12ex^2 - 100x + \frac{50}{\sqrt{x}} - \frac{1}{x^2} + \frac{5}{x^{7/2}} = \frac{3e}{x^4}$$

Score: /3

b. Find $d(f(x))/dx$ where

$$f(x) = \frac{23 + \sqrt{x} - x^5}{\left(1 - \frac{2}{x^3}\right)}$$

$$f'(x) = \frac{\left(\frac{1}{2\sqrt{x}} - 5x^4\right)\left(1 - \frac{2}{x^3}\right) - (23 + \sqrt{x} - x^5)\left(\frac{6}{x^4}\right)}{\left(1 - \frac{2}{x^3}\right)^2}$$

Score: /3