Math 108-01
Summer 2024
Dr. Lily Yen

Quiz 5

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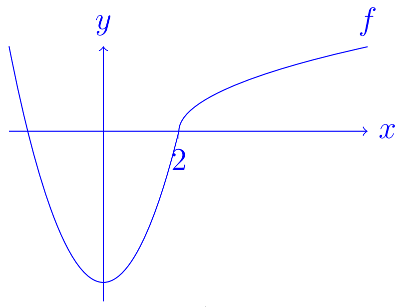
Score: ___/10

Problem 1: Use the limit definition of continuity to find a value c that makes the piece-wise defined function continuous everywhere. Draw your resulting function to check. From the graph, is the function differentiable at $x = 2$?

$$f(x) = \begin{cases} \sqrt{x-2}, & x > 2 \\ x^2 - c, & x \leq 2 \end{cases}$$

Polynomials and roots are continuous. Compositions of continuous functions are continuous. Therefore each piece of f is continuous.
Note that $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} \sqrt{x = 2} = 0$, that $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 - c = 4 - c$, and that $f(2) = 4 - c$. Therefore f is continuous at $x = 2$ (and hence everywhere) if $4 - c = 0$, so $c = 4$.



The graph looks like it has a cusp at $(2, 0)$, so f is likely NOT DIFFERENTIABLE and further analysis bears this out.

Score: /4

Problem 2: Answer the following using derivative rules. Do NOT simplify.

a. Find $g'(x)$ where $g(x) = (5x^2 - 3\sqrt{x} + \pi)\left(3x^4 + \dfrac{1}{x^2} - 100\right)$

$$g'(x) = \left(10x - \frac{3}{2\sqrt{x}}\right)\left(3x^4 + \frac{1}{x^2} - 100\right) + (5x^2 - 3\sqrt{x} + \pi)\left(12x^3 - \frac{2}{x^3}\right)$$
$$= 90x^4 - \frac{81}{2}x^{7/2} + 12\pi x^3 - 1000x + \frac{150}{\sqrt{x}} + \frac{9}{2x^{5/2}} - \frac{2\pi}{x^3}$$

Score: /3

b. Find $d(f(x))/dx$ where

$$f(x) = \frac{2 - \frac{3}{x^4}}{523 + \sqrt{x} - x^6}$$

$$f'(x) = \frac{\frac{12}{x^5}(523 + \sqrt{x} - x^6) - \left(2 - \frac{3}{x^4}\right)\left(\frac{2}{2\sqrt{x}} - 6x^5\right)}{(523 + \sqrt{x} - x^6)^2}$$

Score: /3