Math 108-03
Spring 2026
Dr. Lily Yen

Midterm Two

Show all your work

Name: ______________

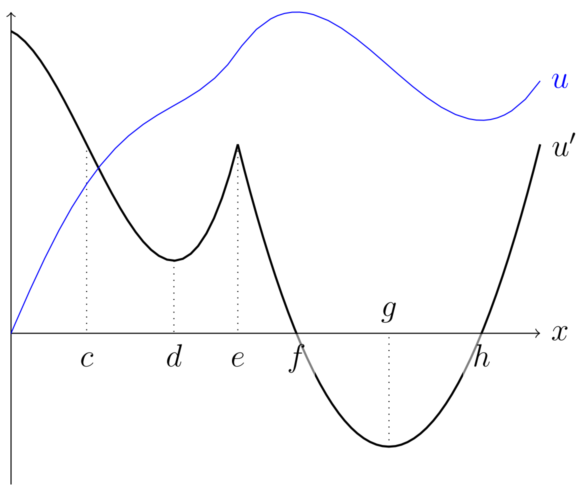
Number: ______________

Signature: ______________

Score: ___/40

No Calculator allowed in this part.

Problem 1: Given the derivative graph of u below, answer the following questions.



a. List the open interval(s) of x where the function u is increasing .

$u'(x) > 0$, so on $(0, f) \cup (h, \infty)$.

b. List open interval(s) where the graph of u is concave up.

The u'' is positive when u' is increasing, so on $(d, e) \cup (g, \infty)$.

c. List all values of x where the graph of u has an inflection point.

Inflection points occur where u'' changes sign, so at $t = d$, $t = e$, and $t = g$.

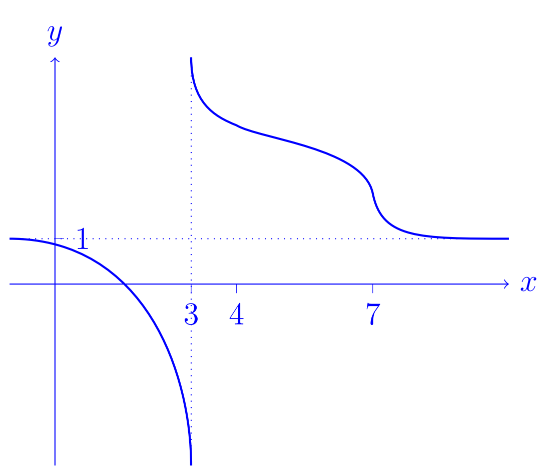
d. Give all values of x where the graph of u has relative extremum. Specify each as a relative maximum or a relative minimum.

The function u has a relative extremum when u' changes sign, so at relative maximum at $x = f$ and a relative minimum at $x = h$ using the first derivative test.

e. Sketch the function u directly on the given coordinate system.

Score: /8

Problem 2: Sketch the graph of a single function satisfying all of the following conditions. • The function is continuous and differentiable for all real numbers except at $x = 3$ where it has a vertical asymptote; • $f'(x) < 0$ everywhere it is defined; • A horizontal asymptote at $y = 1$; • $f''(x) < 0$ on open intervals $(-\infty, 3) \cup (4, 7)$, and • $f''(x) > 0$ on the open interval $(3, 4) \cup (7, \infty)$. Asymptotes must be labelled in your graph.



Score: /4

/12

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Calculators allowed for this part.

Problem 3: Consider the following implicitly defined relationship:

$$y^2(x^2+y^2)=20x^3$$

a. Determine dy/dx.

Taking the derivative of $x^2y^2+y^4=20x^3$ with respect to x yields that $2xy^2+2x^2yy'+4y^3y'=60x^2$, so $(2x^2y+4y^3)y'=60x^2-2xy^2$, so

$$y'=\frac{60x^2-2xy^2}{2x^2y+4y^3}=\frac{30x^2-xy^2}{x^2y+2y^3}$$

Score: /3

b. Find the equation of the tangent line at the point $(1, 2)$.

If $x=1$ and $y=2$, then $y'=\frac{30-2^2}{2+2\cdot 2^3}=\frac{26}{18}=\frac{13}{9}$. Hence the tangent is given by

$$y-2=\frac{13}{9}(x-1) \quad \text{or} \quad y=\frac{13}{9}x+\frac{5}{9} \quad \text{or} \quad 13x-9y+5=0$$

Score: /3

Problem 4: Lake Capilano polluted by bacteria is treated with an antibacterial chemical. After t days, the number N of bacteria per millilitre of water is approximated by

$$N(t)=15\left(\frac{t}{12}-\ln\left(\frac{t}{12}\right)\right)+43$$

for t in $[1, 15]$. Use calculus techniques to answer the following.

a. When during this time will the number of bacteria be a minimum?

$N'(t)=15(\frac{1}{12}-\frac{1}{t})$, which is defined for all t in $[1, 15]$. If $N'(t)=0$, then $\frac{1}{12}=\frac{1}{t}$, so $t=12$.
Now $N(1)=15(\frac{1}{12}-\ln(\frac{1}{12}))+43\approx 81.52$, while $N(12)=58$, and $N(15)=20(\frac{5}{4}-\ln(\frac{5}{4}))+43\approx 58.40$.
Thus the minimum is 58.00 bacteria per ml after 12 days.

b. When during this time will the number of bacteria be a maximum? What is the maximum number?

From the previous part, the maximum is 81.52 bacteria per ml after 1 day.

Score: /5

/11

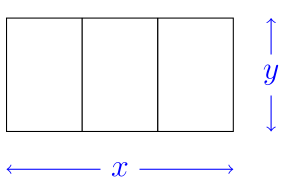
Problem 5: Madame Blueberry is standing on an 8 ft ladder placed against a building trying to paint a window frame blue. The base of the ladder begins slipping away from the building at a rate of 2 ft/min. Find the rate at which the top of the ladder is sliding down the building at the instant when the bottom of the ladder is 3 ft from the base of the building.

Let x be the distance from the base of the wall to the bottom of the ladder, and let y be the distance from the base of the wall to the top of the ladder. Then $x^2 + y^2 = 8^2$, so $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, so $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$.
Now, $\frac{dx}{dt} = 2$, and when $x = 3$, $y = \sqrt{55}$. Hence

$$\frac{dy}{dt} = -\frac{3}{\sqrt{55}} \cdot 2 = -\frac{6}{\sqrt{55}} = -\frac{6\sqrt{55}}{55} \approx -0.8090\,\text{ft/min}$$

Score: /4

Problem 6: The Count of Transylvania is conducting a research project on breeding Transylvanian dragons in captivity. He first must construct suitable pens. He wants a rectangular area with two additional fences across its width, as shown in the diagram. Find the maximum area he can enclose with 3500 m of fencing. Calculus techniques must be used.



The total length of fence is $2x + 4y = 3500$, so $x = 1750 - 2y$. The area is thus $A = xy = (1750 - 2y)y = 1750y - 2y^2$. The domain of $A(y)$ is $[0, 875]$. Therefore $\frac{dA}{dy} = 1750 - 4y$, so $A' = 0$ when $y = \frac{1750}{4} = 437.5\,\text{m}$ and $x = 1750 - 2y = 875.0\,\text{m}$, so $A = xy = 382\,812.5\,\text{m}^2$. Since $\frac{d^2A}{dy^2} = -4 < 0$, this is a maximum by the second derivative test for a concave down function. Since the domain is a closed interval, we need to check endpoints which give 0 area, not where maximum occurs. Without using a derivative test, one can see that $A(y)$ is a downward facing parabola with a unique maximum.

Score: /5

/9

Problem 7: Use Linear Approximation to estimate $\sqrt[3]{8.001}$.

Let $f(x) = \sqrt[3]{x} = x^{1/3}$. Then $f'(x) = \frac{1}{3}x^{-2/3}$, and

$$f(x) \approx f(8) + f'(8)(x-8).$$

Now, $f(8) = \sqrt[3]{8} = 2$, and $f'(8) = \frac{1}{3} \times \frac{1}{2^2} = \frac{1}{12}$, so

$$\sqrt[3]{8.001} = f(8.001) \approx 2 + \frac{1}{12}(8.001 - 8) \approx 2.000\,083\,333\,3$$

Actually, $\sqrt[3]{8.001} \approx 2.000\,083\,329\,9$.

Score: /4

Problem 8: Sociologists have found that crime rates are influenced by temperature. In a prairie town of 100 000 people, the crime rate has been approximated as

$$C(T) = \frac{1}{10}(T-60)^2 + 100$$

where C is the number of crimes per month and T is the average monthly temperature in degrees Fahrenheit. Given that the average temperature for May was 55°F, answer the following.

a. What was the number of crimes in May predicted by the crime rate function?

$$C(55) = 102.5 \approx 103$$

crimes per month in May.

b. Suppose the maximum error for the average temperature in May was 1°F, how would such an error in average temperature affect a change in the crime rate?

Now $C'(T) = \frac{1}{5}(T-60)$, so

$$\frac{\Delta C}{\Delta T} \approx \frac{dC}{dT} = C'(55) = -1$$

so $\Delta C \approx -\Delta T = -1$, so the number of crimes would likely decrease or increase by a single crime.
Actually, $C(55+1) = 101.6$ and $C(55-1) = 103.6$, so the deviation from 102.5 is -0.9 and 1.1, respectively.

Score: /4

/8