Math 108
Spring 2024
Dr. Lily Yen

Quiz Five

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Number: ____________
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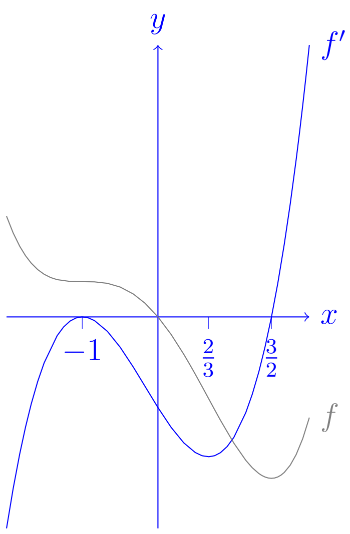
Problem 1: Let the derivative of f be defined by $f'(x) = (2x-3)(x+1)^2$.

a. Draw f'.

b. List all intervals where the original function f is DECREASING.

c. List all inflection points of f.

d. List all intervals where the graph of f is concave UP.



f is decreasing where $f'(x) < 0$ so on $(-\infty, \frac{3}{2})$. And, yes, f is also decreasing at $x = -1$.

f has an inflection point where $f''(x) = 0$. Since $f''(x) = 2(x+1)^2 + (2x-3)2(x+1) = 2\big((x+1)+(2x-3)\big)(x+1) = 2(3x-2)(x+1)$, the inflection points are at $x = \frac{2}{3}$ and at $x = -1$.

f is concave up where $f''(x) > 0$ so on $(-\infty, -1) \cup (\frac{2}{3}, \infty)$.

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Problem 2: Draw the following function including all important features like local extrema, inflection point(s), and asymptotic behaviour. Remember to take appropriate limits to support your claim.

$$y = \frac{x^3}{9-x^2}$$

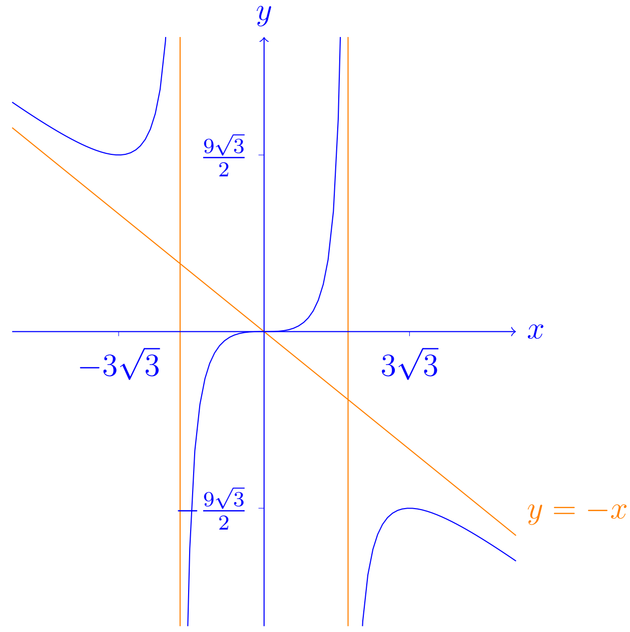
As $x \to \pm\infty$, $y \approx \frac{x^3}{-x^2} = -x$ which gives only the *slope* of the oblique asymptote, but long division,

$$\begin{array}{r|l} & -x, \\ -x^2+9) & x^3 \\ & -x^3+9x \\ \hline & 9x \end{array}$$

yields that $y = -x + \frac{9x}{9-x^2}$ so the y-intercept of the asymptote is actually 0.

$y' = \frac{3x^2(9-x^2)+2x^3x}{(9-x^2)^2} = \frac{x^2(27-x^2)}{(9-x^2)^2}$, so $y' = 0$ if $x = 0$ or $x = \pm\sqrt{27} = \pm 3\sqrt{3}$, and y' is undefined if $x = \pm 3$.

$y'' = \frac{(54x-4x^3)(9-x^2)^2+4(27x^2-x^4)(9-x^2)x}{(9-x^2)^4} = \frac{(54x-4x^3)(9-x^2)+4(27x^2-x^4)x}{(9-x^2)^3} = \frac{18x^3+486x}{(9-x^2)^3} = \frac{18x(x^2+27)}{(9-x^2)^3}$, so $y'' = 0$ if $x = 0$, and y'' is undefined if $x = \pm 3$.



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