Math 108
Spring 2024
Dr. Lily Yen

Quiz 5

Show all your work

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Number: ____________________
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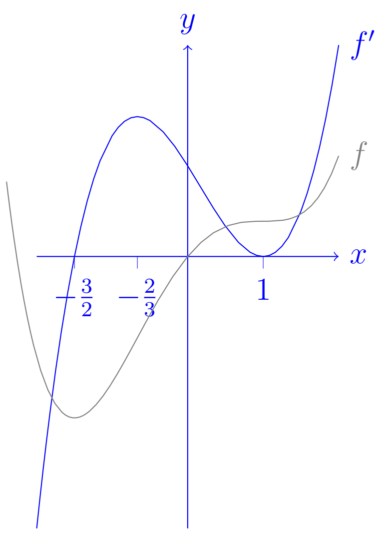
Problem 1: Let the derivative of f be defined by $f'(x) = (2x+3)(x-1)^2$.

a. Draw f'.

b. List all intervals where the original function f is INCREASING.

c. List all inflection points of f.

d. List all intervals where the graph of f is concave DOWN.



f is increasing where $f'(x) > 0$ so on $(-\frac{3}{2}, \infty)$. And, yes, f is also increasing at $x = 1$.

f has an inflection point where $f''(x) = 0$. Since $f''(x) = 2(x-1)^2 + (2x+3)2(x-1) = 2\big((x-1)+(2x+3)\big)(x-1) = 2(3x+2)(x-1)$, the inflection points are at $x = -\frac{2}{3}$ and at $x = 1$.

f is concave down where $f''(x) < 0$ so on $(-\frac{2}{3}, 1)$.

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Problem 2: Draw the following function including all important features like local extrema, inflection point(s), and asymptotic behaviour. Remember to take appropriate limits to support your claim.

$$y = \frac{x^3}{4-x^2}$$

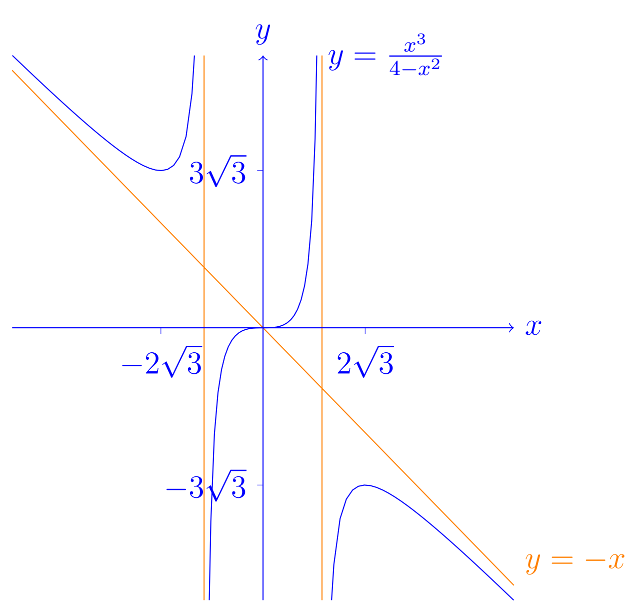
As $x \to \pm\infty$, $y \approx \frac{x^3}{-x^2} = -x$ which gives only the *slope* of the oblique asymptote, but long division,

$$\begin{array}{r|l} & -x, \\ -x^2+4 & x^3 \\ & -x^3+4x \\ \hline & 4x \end{array}$$

yields that $y = -x + \frac{4x}{4-x^2}$ so the y-intercept of the asymptote is actually 0.

$y' = \frac{3x^2(4-x^2)+2x^3x}{(4-x^2)^2} = \frac{x^2(12-x^2)}{(4-x^2)^2}$, so $y' = 0$ if $x = 0$ or $x = \pm\sqrt{12} = \pm 2\sqrt{3}$, and y' is undefined if $x = \pm 2$.

$y'' = \frac{(24x-4x^3)(4-x^2)^2+4(12x^2-x^4)(4-x^2)x}{(4-x^2)^4} = \frac{(24x-4x^3)(4-x^2)+4(12x^2-x^4)x}{(4-x^2)^3} = \frac{8x^3+96x}{(4-x^2)^3} = \frac{8x(x^2+12)}{(4-x^2)^3}$, so $y'' = 0$ if $x = 0$, and y'' is undefined if $x = \pm 2$.



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