Math 108
Spring 2024
Dr. Lily Yen

Quiz Four

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Number: ____________
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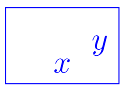
Score: ___/10

Problem 1: The length of a rectangle is shrinking at a rate of 2 cm/min while the width of the rectangle is increasing at a rate of 1 cm/min. Find the rate at which the area of the rectangle changes when the length is 15 cm and the width is 10 cm.

Given: 

$\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 1$.
The area is $A = xy$, so $\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$, so when $x = 15$ and $y = 10$,

$$\frac{dA}{dt} = -2 \cdot 10 + 15 \cdot 1 = -5\,\text{cm}^2/\text{min}$$

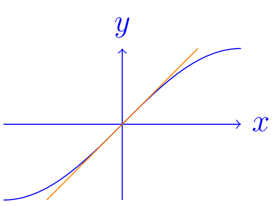
So the area of the rectangle is decreasing at $5\,\text{cm}^2$ per minute.

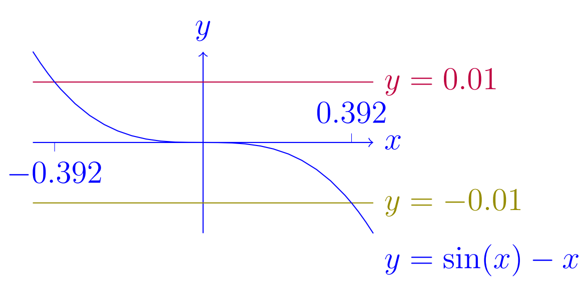
Score: /3

Problem 2: Use the technique of linear approximation to estimate $\sin(0.02)$ within 0.01 accuracy. State clearly your $f(x)$ and anchor point a before applying Linear Approximation Formula. Draw the graph and specify the interval for x around $x = a$ where accuracy is attained.

$f(x) \approx f(a) + f'(a)(x - a)$. Here $a = 0$ and $f(x) = \sin(x)$, so $f'(x) = \cos(x)$ and

$$\sin(x) \approx \sin(0) + \cos(0)(x - 0) = x, \quad \text{so } \sin(0.02) \approx 0.02$$



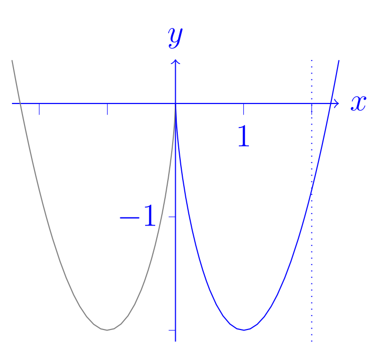


Score: /4

Problem 3: For the following function, find the absolute extrema over the specified interval and state where those values occur. Draw the function.

$$f(x) = x^2 - 3x^{2/3}, \quad x \in [0, 2]$$

$f'(x) = 2x - 3 \cdot \frac{2}{3}x^{-1/3} = 2x - 2x^{-1/3} = 2(x - x^{-1/3})$, so $f'(x)$ is undefined when $x = 0$, and $f'(x) = 0$ when $x = x^{-1/3}$, so $x^3 = x^{-1}$, so $x^4 = 1$, so $x = \pm 1$. Since $x \in [0, 2]$, that leaves $x = 0$ and $x = 1$ as the critical values. Evaluating at the critical values and the endpoints yields $f(0) = 0$, $f(1) = -2$, and $f(2) = 4 - 3(2)^{2/3} \approx -0.7622$. Therefore the absolute minimum is $(1, -2)$ and the absolute maximum is $(0, 0)$.



Score: /3