Name: ____________________

Number: ____________________

Signature: ____________________

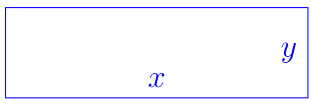
Math 108
Spring 2024
Dr. Lily Yen

Quiz 4

Show all your work

Score: ___/10

Problem 1: The width of a rectangle is shrinking at a rate of 1 cm/min while the length of the rectangle is increasing at a rate of 3 cm/min. Find the rate at which the area of the rectangle changes when the width is 12 cm and the length is 40 cm.

Given: 

$\frac{dx}{dt} = 3$, $\frac{dy}{dt} = -1$.

The area is $A = xy$, so $\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$, so when $x = 40$ and $y = 12$,

$$\frac{dA}{dt} = 3 \cdot 12 + 40 \cdot (-1) = -4\,\text{cm}^2/\text{min}$$

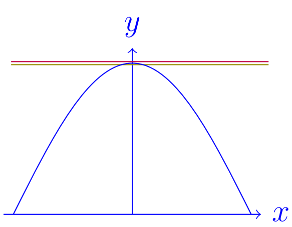
So the area of the rectangle is decreasing at $4\,\text{cm}^2$ per minute.

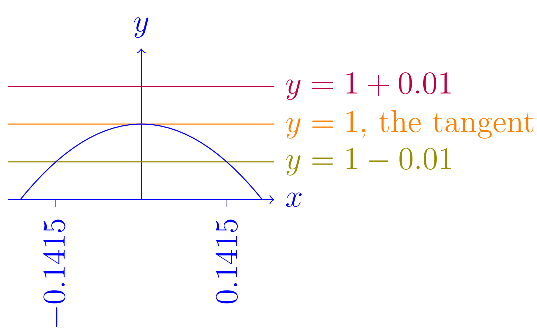
Score: /3

Problem 2: Use the technique of linear approximation to estimate cos(0.03) within 0.01 accuracy. State clearly your $f(x)$ and anchor point a before applying Linear Approximation Formula. Draw the graph and specify the interval for x around $x = a$ where accuracy is attained.

$f(x) \approx f(a) + f'(a)(x - a)$. Here $a = 0$ and $f(x) = \cos(x)$, so $f'(x) = -\sin(x)$ and

$$\cos(x) \approx \cos(0) - \sin(0)(x - 0) = 1, \quad \text{so } \cos(0.03) \approx 1$$



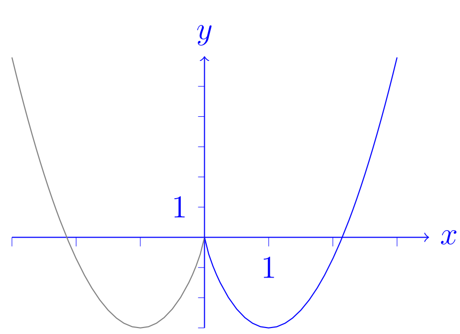


Score: /4

Problem 3: For the following function, find the absolute extrema over the specified interval and state where those values occur. Draw the function.

$$f(x) = 2x^2 - 5x^{4/5}, \quad x \in [0, 3]$$

$f'(x) = 2 \cdot 2x - 5 \cdot \frac{4}{5}x^{-1/5} = 4x - 4x^{-1/5} = 4(x - x^{-1/5})$, so $f'(x)$ is undefined when $x = 0$, and $f'(x) = 0$ when $x = x^{-1/5}$, so $x^5 = x^{-1}$, so $x^6 = 1$, so $x = \pm 1$. Since $x \in [0, 3]$, that leaves $x = 0$ and $x = 1$ as the critical values. Evaluating at the critical values and the endpoints yields $f(0) = 0$, $f(1) = -3$, and $f(3) = 18 - 5(3)^{4/5} \approx 5.959$. Therefore the absolute minimum is $(1, -3)$ and the absolute maximum is $(3, 5.959)$.



Score: /3