

Math 108  
Spring 2017  
Dr. Lily Yen

# Test 2

Show all your work

Name: \_\_\_\_\_  
Number: \_\_\_\_\_  
Signature: \_\_\_\_\_  
Score: \_\_\_\_/45

**No Calculator allowed in this part.**

**Problem 1:** Use the rules of differentiation to find the derivative of each of these functions. Perform any obvious SIMPLIFICATIONS—coefficients, exponents, etc. DO NOT leave negative exponents in your answer.

a. Differentiate  $f(x) = \frac{19}{x} + \sqrt{4e} - 5^{3x}$ .

b. Find  $g'(t)$  where  $g(t) = 2 \ln(3t^6) - \cos(t^2) + \frac{\pi}{\sqrt[3]{t^2 - 5}}$ .

Score: /4

**Problem 2:** Determine the following derivatives using differentiation rules. Do NOT simplify.

a.  $f(x) = (e^{7x+2} - \sqrt{x^5 + 1})^{3/5} (\log_5 x - \sin(4x))$

Score: /3

b.  $v(t) = \frac{te^t}{\ln(t^2 - 3) + \tan(3t^{2/3} - 5)}$

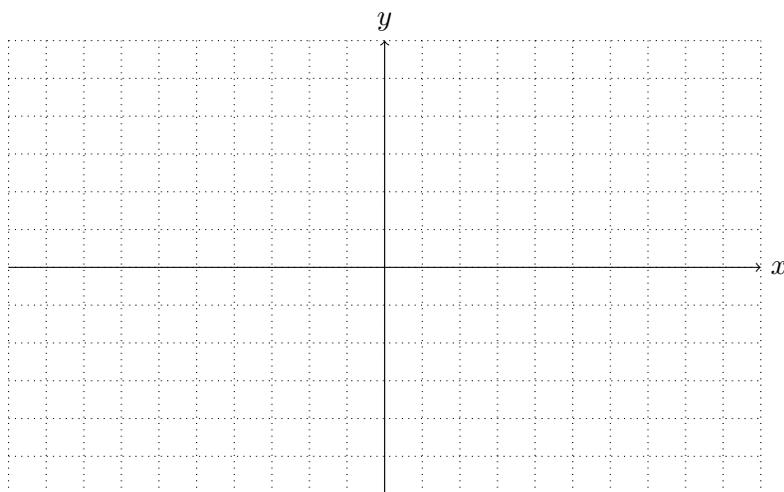
Score: /4

**Problem 3:** Given that  $f(x) = e^{2x} + \sin(x)$ , first find the 4th derivative of  $f$ , namely,  $f^{(4)}(x)$ . Then find a general formula for the  $4n + 1$ st derivative of  $f$  with respect to  $x$ .

Score: /4

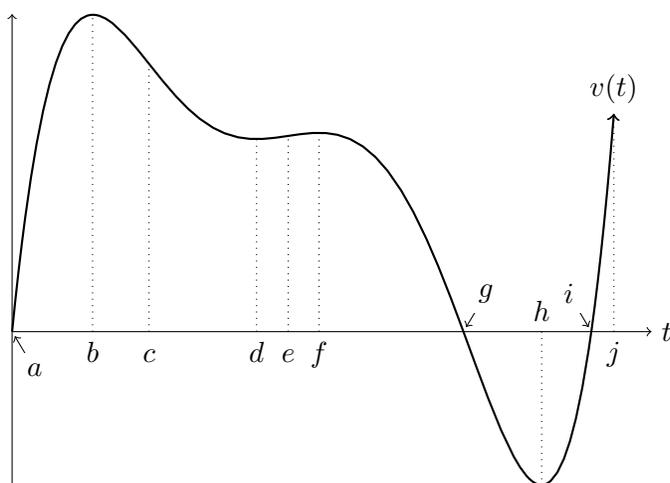
**Problem 4:** Sketch the graph of a single function that has all the properties listed.

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|--|---|
| <ul style="list-style-type: none"> <li>• Continuous and differentiable everywhere except at <math>x = 3</math> where it has a vertical asymptote.</li> <li>• Three <math>x</math>-intercepts, at <math>x = -6, 2, 7</math>.</li> <li>• A <math>y</math>-intercept at <math>y = 4</math>.</li> <li>• A horizontal asymptote at <math>y = -1</math>.</li> <li>• <math>f'(x) &lt; 0</math> on intervals <math>(3, \infty)</math> and <math>(-2, 3)</math>.</li> </ul> | <ul style="list-style-type: none"> <li>• <math>f'(x) &gt; 0</math> on interval <math>(-\infty, -2)</math>.</li> <li>• <math>f''(x) &gt; 0</math> on the intervals <math>(-\infty, -4)</math>, and <math>(3, \infty)</math></li> <li>• <math>f''(x) &lt; 0</math> on the interval <math>(-4, 3)</math>.</li> <li>• <math>f'(-2) = 0</math>.</li> <li>• An inflection point at <math>(-4, 3)</math>.</li> </ul> |
|--|---|



Score: /5

**Problem 5:** Your remote controlled car is moving along a straight stretch of road. The velocity of the car is given by the graph shown. Answer the following questions.



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|--|---|
| <p>a. When is the absolute minimum velocity attained?</p> <p>b. When is the absolute maximum velocity attained in the time interval <math>(a, j)</math>?</p> <p>c. When does your car begin to return to where it started?</p> <p>d. List open intervals for <math>t</math> in <math>(a, j)</math> where</p> | <p>the car has a positive acceleration.</p> <p>e. In the interval <math>(0, \infty)</math>, what time was the acceleration at its least?</p> <p>f. List times when the position function related to this velocity function has an inflection point.</p> |
|--|---|

Score: /6

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**Calculators allowed in this part.**

**Problem 6:** The age/mass relationship of male Arctic foxes caught in Svalbard, Norway, can be estimated by the function

$$M(t) = 3583e^{-e^{-0.020(t-66)}},$$

where  $t$  is the age of the fox in days and  $M(t)$  is the mass of the fox in grams.

a. Estimate the mass of a male Arctic fox that is 250 days old.

b. Find and interpret  $M'(100)$ .

c. Estimate the age of a male fox when it has reached 50% of its maximum mass.

d. Use your graphing calculator to graph  $M(t)$  and then describe the growth pattern.

Score: /5

**Problem 7:** Suppose that the profit (in hundreds of dollars) from selling  $x$  units of a product is given by  $P(x) = \frac{x^2}{3 + 5x}$ .

a. Find and interpret the marginal profit when 20 units are sold. State your marginal profit function.

b. Find and interpret the marginal AVERAGE profit when 10 units are sold.

c. Does this profit function have a point of diminishing returns? Show your work to support your conclusion.

Score: /5

**Problem 8:** Under the scenario that the fertility rate in the European Union (EU) remains at 1.8 until 2020, when it rises to replacement level, the predicted population (in millions) of the 15 member countries of the EU can be approximated over the next century by

$$P(t) = 325 + 7.475(t + 10)e^{-(t+10)/20},$$

where  $t$  is the number of years since 2000.

- a. In what year is the population predicted to be largest? What is the population predicted to be in that year? You must use calculus techniques to support your claim. Graph relevant function(s).

- b. In what year is the population declining most rapidly? Again, use calculus techniques. Graph relevant function(s).

- c. What is the population approaching as time goes on? Show your steps.

Score: /9