

Test 1

Show all your work

Name: _____

Score: ____/39

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞ , and $-\infty$ where appropriate.

a. $\lim_{x \rightarrow 16} \frac{4 - \sqrt{x}}{x - 16} =$

Score: /2

b. $\lim_{y \rightarrow 2^-} \frac{y^2 + y - 6}{|y - 2|} =$

Score: /2

c. $\lim_{a \rightarrow -\infty} \frac{2a^3 - a - 43}{5a^2 + a + 1} =$

Score: /2

Problem 2:

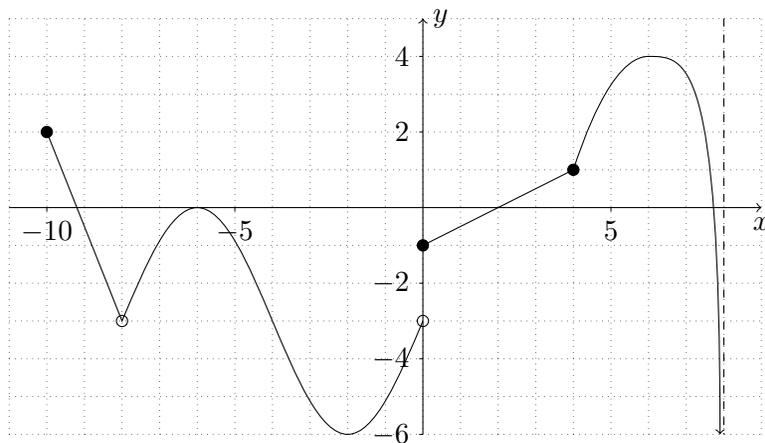
a. State the definition of continuity of a function f at a point, say $x = c$. Also include all three conditions satisfied by f implied by the definition.

b. Find the value of the constant k that makes the function continuous.

$$f(x) = \begin{cases} x^3 + k, & x \leq 3 \\ kx - 5, & x > 3 \end{cases}$$

Score: /4

Problem 3: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞ , and $-\infty$ where appropriate.



a. Find the domain of f in as few intervals as possible.

b. Find the range of f in as few intervals as possible.

c. $f(0) =$

d. $\lim_{h \rightarrow -8} f(h) =$

e. $\lim_{x \rightarrow 0} f(x) =$

f. $\lim_{x \rightarrow \pi} \frac{f(x) - f(\pi)}{x - \pi} =$

g. $\lim_{x \rightarrow 8^-} f(x) =$

h. State all value(s) of x in the domain where f is discontinuous.

i. List the x value(s) in the interval $(-8, 8)$ for which f is (defined and) continuous, but $f'(x)$ is undefined.

j. Use the tangent line to estimate the value of $f'(-2)$.

k. In the same grid above, graph $y = f'(x)$ for the interval $(-10, 0)$.

Score: /12

Problem 4: Given $f(x) = \frac{3}{x}$, use the definition of the derivative to find $f'(x)$ and the equation of the tangent line at $x = 1$.

Score: /4

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Calculators allowed from here on.

Problem 5: The price of a barrel of West Texas Intermediate crude oil from July 2014 to January 2015 is given bimonthly in the following table.

| July | | Aug | | Sep | | Oct | | Nov | | Dec | | Jan | |
|-------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 1 | 15 | 1 | 15 | 1 | 15 | 1 | 15 | 1 | 15 | 1 | 15 | 1 | 15 |
| \$101 | \$97 | \$95 | \$92 | \$93 | \$92 | \$88 | \$82 | \$77 | \$75 | \$67 | \$56 | \$52 | \$46 |

- a. Examine and draw a scatter plot and determine the best quadratic model to fit the data. State your model correct to 3 decimal places. Be sure to define your variables.

Score: /2

- b. Use your model to predict the price of a barrel of crude oil in the beginning of March, 2015, and comment on the accuracy of this prediction. Give 2 decimal place accuracy.

Score: /1

- c. According to your model, what was the price of crude oil on January 1st, 2015? Find the error of the estimate. Give 2 decimal place accuracy.

Score: /2

- d. According to your model, when will the price fall below forty dollars a barrel? Comment on the accuracy of this prediction.

Score: /1

Problem 6: When glucose is infused into a person's bloodstream at a constant rate of c grams per minute, the glucose is converted and removed from the bloodstream at a rate proportional to the amount present. The amount of glucose in grams in the bloodstream at time t (in minutes) is given by

$$g(t) = \frac{c}{a} + \left(g_0 - \frac{c}{a}\right) e^{-at},$$

where a is a positive constant. Assume $g_0 = 0.08$ g, $c = 0.1$ g/min, and $a = 1.3$ min⁻¹.

- Find the total change in the amount of glucose in the bloodstream in the first 10 minutes. Give 5 decimal place accuracy.
- Find the average rate of change in the amount of glucose in the bloodstream over the first 10 minutes. Give 5 decimal place accuracy.
- Estimate the instantaneous rate of change for the amount of glucose in the bloodstream 1 minute after initial infusion. Use the chart below to document your estimate from average rate of change (ARC) to its limiting value. Give 4 decimal place accuracy.

| interval | ARC | interval | ARC |
|----------|-----|----------|-----|
| | | | |
| | | | |
| | | | |
| | | | |

- When is the amount of glucose decreasing most rapidly? To answer the question, you may want to use your calculator to sketch the graph of g .

e. Interpret $g(4) =$ in non-mathematical terms. Include units. Give 5 decimal place accuracy.

f. Interpret $g'(4) =$ in non-mathematical terms. Include units. Give 4 decimal place accuracy.

Score: /7