Math 108
Fall 2018
Dr. Lily Yen

Test 1

Show all your work

Name: ____________________

Score: ___/40

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\lim_{a\to-\infty} \frac{2+8a^2-3a}{7a-5-11a^2} =$ $\boxed{-\frac{8}{11}}$

$\lim_{a\to-\infty} \frac{2+8a^2-3a}{7a-5-11a^2} = \lim_{a\to-\infty} \frac{a^2(\frac{2}{a^2}+8-\frac{3}{a})}{a^2(\frac{7}{a}-\frac{5}{a^2}-11)} = \lim_{a\to-\infty} \frac{\frac{2}{a^2}+8-\frac{3}{a}}{\frac{7}{a}-\frac{5}{a^2}-11} = \frac{8}{-11} = -\frac{8}{11}$

Score: /2

b. $\lim_{h\to0} \frac{(x+h)^2-3-(x^2-3)}{h} =$ $\boxed{2x}$

$\lim_{h\to0} \frac{(x+h)^2-3-(x^2-3)}{h} = \lim_{h\to0} \frac{x^2+2xh+h^2-3-x^2+3}{h} = \lim_{h\to0} \frac{2xh+h^2}{h} = \lim_{h\to0} 2x+h = 2x$

Score: /2

c. $\lim_{x\to-2^-} \frac{x^2-x-6}{x^2+4x+4} =$ $\boxed{\infty}$

$\lim_{x\to-2^-} \frac{x^2-x-6}{x^2+4x+4} = \lim_{x\to-2^-} \frac{(x-3)(x+2)}{(x+2)^2} = \lim_{x\to-2^-} \frac{x-3}{x+2} = +\infty$

Score: /2

d. $\lim_{x\to9^+} \frac{\sqrt{x}+3}{9-x} =$ $\boxed{-\infty}$

Since $\lim_{x\to9^+} \sqrt{x}+3 = \sqrt{9}+3 = 6$ while $\lim_{x\to9^+} 9-x = -0$, it follows that $\lim_{x\to9^+} \frac{\sqrt{x}+3}{9-x} = -\infty$

Score: /2

e. If $\lim_{x\to5} f(x) = -2$ and $\lim_{x\to5} g(x) = 3$, find $\lim_{x\to5} \frac{4g(x)-1}{7+2f(x)} =$ $\boxed{11/3}$

$\lim_{x\to5} \frac{4g(x)-1}{7+2f(x)} = \frac{4\cdot3-1}{7+2\cdot(-2)} = \frac{11}{3}$

Score: /2

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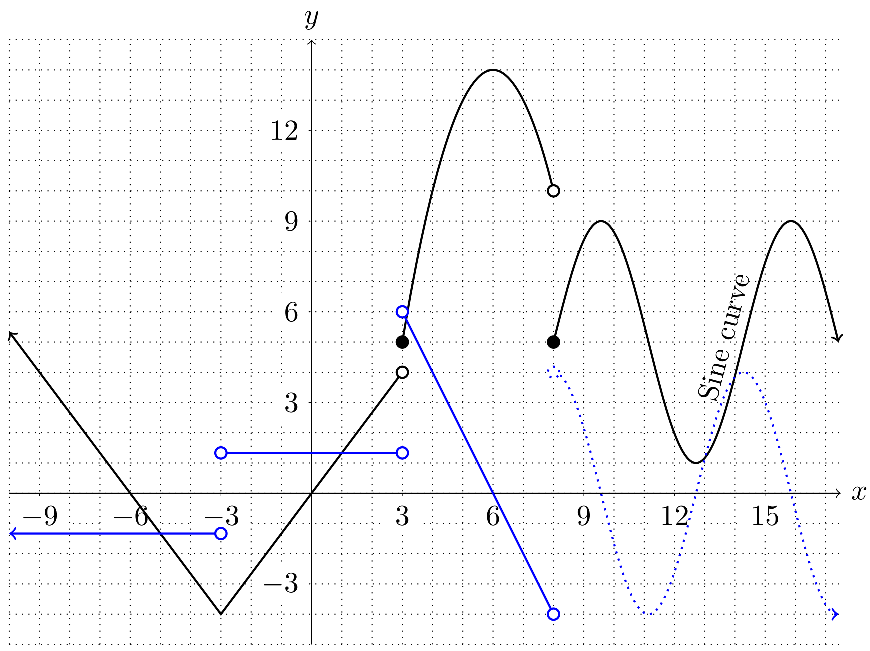
Calculators allowed from here on.

Problem 2: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. Express in as few intervals as possible the range of f.

$[-4, \infty)$ assuming that the ray on the left continues all the way up.
Otherwise, $[-4, 14]$.

b. Express in as few intervals as possible where f is continuous in $(-\infty, \infty)$.

$(-\infty, 3) \cup (3, 8) \cup (8, \infty)$

c. $\lim_{x\to 3^-} f(x) =$ **4**

d. $\lim_{x\to 8} f(x) =$ **DNE**

e. $\lim_{x\to\infty} f(x) =$ **DNE**

f. $\lim_{h\to 0} \dfrac{f(e+h) - f(e)}{h} =$ **4/3**

g. $f'(-3) =$ **DNE**

h. $\lim_{x\to 6} \dfrac{f(x) - f(6)}{x - 6} =$ **0**

i. In the same grid above, graph $y = f'(x)$ for the interval $(-\infty, 8)$.

Score: /10

Problem 3: Use the definition of continuity to find a value k that makes the piece-wise defined function continuous everywhere.

$$f(x) = \begin{cases} 2x^2 + 5x + k, & x \le -3 \\ kx, & x > -3 \end{cases}$$

Since each piece is a polynomial, f is continuous for $x \ne -3$.
Now, $\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} 2x^2 + 5x + k = 3 + k$ and $\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} k * x = -3k$. Moreover, $f(-3) = 3 + k$. For f to be continuous at $x = -3$ these three must be equal, so $3 + k = -3k$, so $4k = -3$, so $k = -\frac{3}{4}$.

Score: /4

Problem 4: Use correct notation, show all steps and leave your answer in simplified form.

a. Use the limit definition of the derivative to find the derivative of $f(x) = \sqrt{x} - 1$.

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - 1) - (\sqrt{x} - 1)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}
\end{aligned}
$$

b. For the point $(4, 1)$, find an equation of the tangent line to f.

Since $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$, the tangent line is

$$y - 1 = \frac{1}{4}(x - 4) \quad \text{or} \quad y = \frac{1}{4}x \quad \text{or} \quad x - 4y = 0$$

Score: /4

Problem 5: Assume that the equation $A(x) = -0.125x^2 + 2x + 1.125$ models attendance at Silver Star for Black Panther for the xth week ($A(x)$ in thousands of guests).

a. During which week will Black Panther attain its highest attendance at Silver Star?

Your graphing calculator finds that the maximum occurs when $x = 8$, so in the eighth week.

b. Find the average rate of change in attendance over the first 3 weeks.

$$\frac{A(3) - A(0)}{3 - 0} = 1.6250 \,\text{thousand guests/week}$$

c. Estimate the instantaneous rate of change in attendance for the fifth week. Use the chart below to document your estimate from average rate of change (ARC) to its limiting value.

interval	ARC	interval	ARC

With the fixed point at $x = 5$, let $y_1 = A(x)$, and $y_2 = \frac{y_1(x) - y_1(5)}{x-5}$ and set up intervals where $x = 5$ is at one end of the interval, and the interval length decreases. The limit should be 0.7500 thousandguests/week.

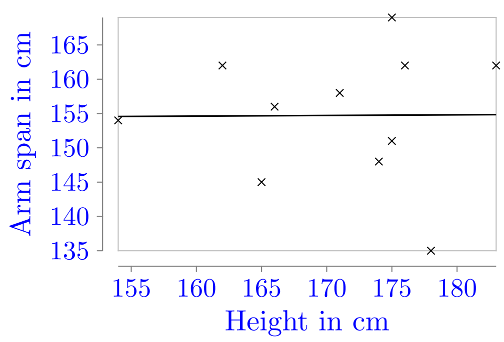
Score: /5

Problem 6: For Nikolaj's physics class, the height and arm span of each student was recorded. Below is the data set of eleven students.

Height (cm):	174	183	171	162	178	166	154	176	165	175	175
Arm span (cm):	148	162	158	162	135	156	154	162	145	169	151

Use the given data to answer the following questions:

a. Draw a scatter plot. Provide dimensions of the window and label your axes.



Score: /2

b. Use linear regression to find a model to fit your plot. Report your model to six decimal places.

$$y = 0.009\,359x + 153.128\,526$$

Score: /2

c. According to your model, what is the arm span of a student with a height of 190 cm? Comment on the reliability of your answer.

If $x = 190$, then $y \approx 155$ cm.
Extrapolation is always dubious, and in this case the data is very scattered.

Score: /1

d. According to your model, what is the predicted height for a student with an arm span of 160 cm? Comment on the reliability of your answer.

When you are unable to see the intersection point with $y_2 = 160$ because your window is too small, you can solve by hand to get 734 cm.
If $y = 160$, then $y \approx 734$ cm. This is clearly ridiculous.

Score: /1

e. Comment on the reliability of your linear regression model relative to the scatter plot.

This data is much too scattered to make a model.

Score: /1