Math 108
Fall 2017
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Test 3

Show all your work

Name: ____________________

Score: ___/38

One calculator (TI-83 or 84) is allowed for this test.

Problem 1: In the Little Campbell River in South Surrey, the number of salmon swimming upstream to spawn is approximated by

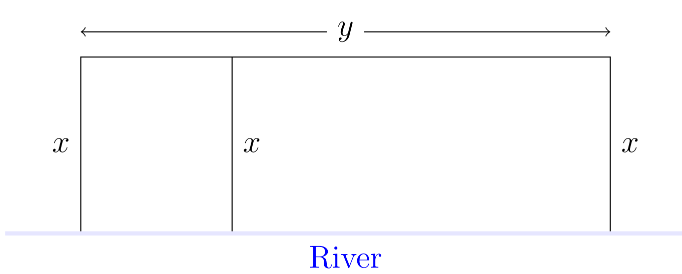
$$S(x) = -x^3 + 3x^2 + 360x + 5000, \quad 6 \le x \le 20,$$

where x represents the temperature of the water in degrees Celsius. Find the water temperature that produces the maximum number of salmon swimming upstream. Your work needs to include calculus techniques.

$S'(x) = -3x^2 + 6x + 360$, so $S'(x) = 0$ when $x = \frac{-6\pm\sqrt{4356}}{-6}\frac{-6\pm 66}{-6} = -10, 12$.
Reject $x = -10$ because it is out of the domain $[6, 20]$.
Since $S(6) = 7052$, $S(12) = 8024$, and $S(20) = 5400$, the optimal temperature is 12 °C.
The second derivative test shows that $S''(x) = -6x + 6$ and $S''(12) = -66 < 0$, so concave down at $x = 12$, or $x = 12$ is a relative maximum. Boundary value checks confirms that it is also an absolute maximum.

Score: /5

Problem 2: A Rocha directors have 1400 m of fencing and would like to enclose a rectangular field bordering Little Campbell River, with no fencing needed along the river. Suppose the fencing material also needs to subdivide the field into two both of which with water access, find the dimensions of the entire rectangular field that maximizes its area.



Let the the fence parallel to the river have length y and the other parts each have length x as in the figure. Then $3x + y = 1400$, so $y = 1400 - 3x$, so the area is
$A = xy = x(1400 - 3x) = 1400x - 3x^2$. Since $A' = 1400 - 6x = 0$ when $x = \frac{700}{3}$ and
$A'' = -6 < 0$, the maximal area is attained when $x = \frac{700}{3} = 233.3$ m and
$y = 1400 - 3x = 700$ m. In that case, the area is $163\,333\ \text{m}^2$.

Score: /6

Problem 3: Burnaby Capital Hill Girl Guides have been collecting aluminum cans for recycling. The group has already collected 6000 kg of cans, for which they could currently receive \$15 per hundred kilograms. The Girl Guides can continue to collect cans at the rate of 200 kg per day. However, a glut in the aluminum market has caused the recycling company to announce that it will lower its price, starting immediately, by \$0.31 per hundred kilograms per day. The Girl Guides can make only one trip to the recycling centre. Find the best time for the trip. What total income will be received?

Say they go to the recycling centre after x days. Then their quantity is $60 + 2x$ (in hundreds of kilograms of cans), and the price is $15 - 0.31x$ (in dollars per hundred kg). The revenue is therefore $r = pq = (15 - 0.31x)(60 + 2x) = 900 + 11.4x - 0.62x^2$, so $r' = 11.4 - 1.24x$. Thus $r' = 0$ when $x = \frac{11.4}{1.24} = 9.2$.
The second derivative test shows also that $r'' = -1.24 < 0$, so concave down. Thus, $x = 9.2$ is a local max for x in $[0, 1500/31]$. Boundary values give 0 for $r(x)$.
Thus, the Girl Guides should recycle after nine days in which case they will earn \$952.38.

Score: /6

Problem 4: Researchers have found a correlation between respiratory rate and body mass in the first three years of life. This correlation can be expressed by the function

$$\log R(w) = 1.83 - 0.43 \log(w),$$

where w is the body mass (in kilograms) and $R(w)$ is the respiratory rate (in breaths per minute) for children. Use implicit differentiation to find $R'(w)$. Express $R'(w)$ as a function of w only.

$$\frac{R'(w)}{R(w)\ln(10)} = 0 - \frac{0.43}{w\ln(10)},$$

so

$$R'(w) = -0.43\frac{R(w)}{w} = -\frac{0.43 \times 10^{1.83}}{w^{1.43}}$$

Score: /4

Problem 5: The energy cost of horizontal locomotion as a function of the body mass of a lizard is given by

$$E(m) = 26.5m^{-0.34},$$

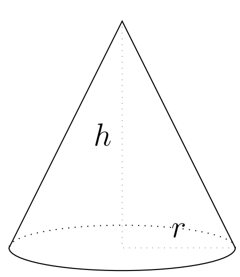
where m is the mass of the lizard (in kilograms) and E is the energy expenditure (in kcal/kg/km). Suppose that the mass of a 2 kg lizard is increasing at a rate of 0.06 kg/day. Find the rate at which the energy expenditure is changing with respect to time.

$$\frac{dE}{dt} = 26.5 \cdot (-0.34)m^{-1.34} \cdot \frac{dm}{dt} = -9.01m^{-1.34} \cdot \frac{dm}{dt}.$$

With the given data, $\frac{dE}{dt} = -9.01(2)^{-1.34} \cdot 0.06 = -0.214\,\text{kcal/kg/km/day}$.
At the time the lizard body mass is at 2 kg, the rate of energy expenditure is decreasing at 0.2135 kcal/kg/km/day.

Score: /4

Problem 6: A sand storage tank used by the highway department for winter storms is leaking. As the sand leaks out, it forms a conical pile. The radius of the base of the pile increases at the rate of 1.54 cm/minute. The height of the pile is always twice the radius of the base. Find the rate at which the volume of the pile is increasing at the instant the radius of the base is 10 cm.



The volume is $V = \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$, so $\frac{dV}{dt} = 2\pi r^2 \cdot \frac{dr}{dt}$.
With the given data, $\frac{dV}{dt} = 2\pi(10\,\text{cm})^2 \cdot 1.54\,\text{cm/min} = 968\,\text{cm}^3/\text{min}$.
The volume increases at $968\,\text{cm}^3/\text{min}$ when the radius of the cone at the base is at 10 cm.

Score: /5

Problem 7: Use differentials to approximate $e^{-0.002}$.

$\frac{d}{dx}e^x = e^x$, so near $x = 0$ you have that $e^x \approx e^0 + e^0(x - 0) = 1 + x$. Therefore $e^{-0.002} \approx 1 - 0.002 = 0.998$.
This is actually a very good estimate compared to the graphing calculator's $e^{-0.002} \approx 0.9980019987$.

Score: /4

Problem 8: A brain tumour is approximately spherical in shape. If the radius of the tumour changes from 14 mm to 16 mm, find the approximate change in volume. Hint: the volume of a sphere of radius r is $4\pi r^3/3$.

Since $V = \frac{4}{3}\pi r^3$, it follows that $\frac{dV}{dr} = 4\pi r^2$, so $\Delta V \approx 4\pi r^2 \cdot \Delta r$.
In this case, $r = 14$ and $\Delta r = 2$, so $\Delta V = 4\pi(14)^2 \cdot 2 = 1568\pi \approx 4926\,\text{mm}^3$
The volume would increase approximately $4926\,\text{mm}^3$.
Actually, $V(16) - V(14) = \frac{5408}{3}\pi \approx 5663\,\text{mm}^3$.

Score: /4