Math 108
Fall 2017
Dr. Lily Yen

Test 2

Show all your work

Name: _____

Score: ___/40

No Calculator allowed in this part.

Problem 1: Use the rules of differentiation to find the derivative of each of these functions. Perform any obvious SIMPLIFICATIONS—coefficients, exponents, etc.

a. Find $\dfrac{d}{dx}\left(\dfrac{7}{x^5} - e^{3x} + \pi^6\right)$.

Score: /2

b. Differentiate $f(x) = \ln(x^e - 3\sqrt{x}) - \dfrac{2}{\sqrt[3]{6-x}}$.

Score: /3

Problem 2: Determine the following derivatives using differentiation rules. Do NOT simplify.

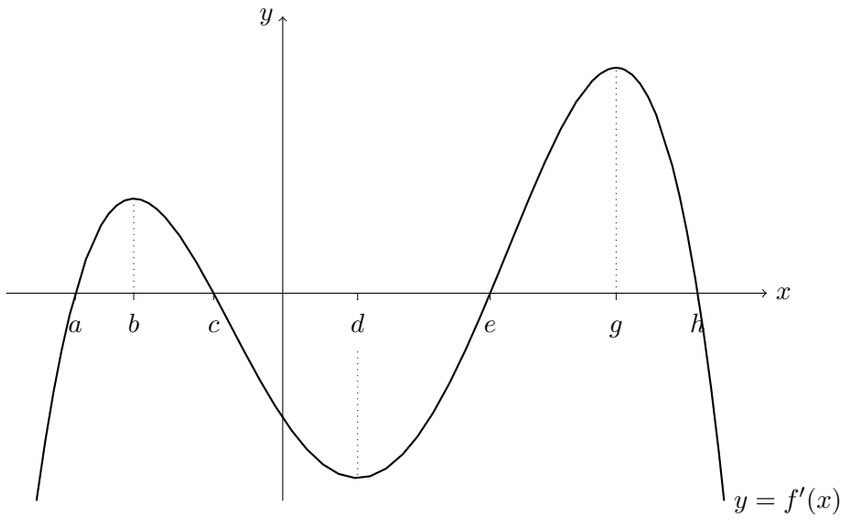
a. Find the derivative of $g(t) = \dfrac{\sin^2(t^3 + 1/t)}{\cot^3(\sqrt{1-3t})}$

Score: /4

b. $f(x) = \left(5^{3x^2} + \sqrt[4]{5x-2}\right)^6 \left(\log(x^3 + \sqrt{ex})\right)^4$

Score: /4

Problem 3: Using the given graph of f', answer the following questions for f.



a. List open intervals of decrease for f.

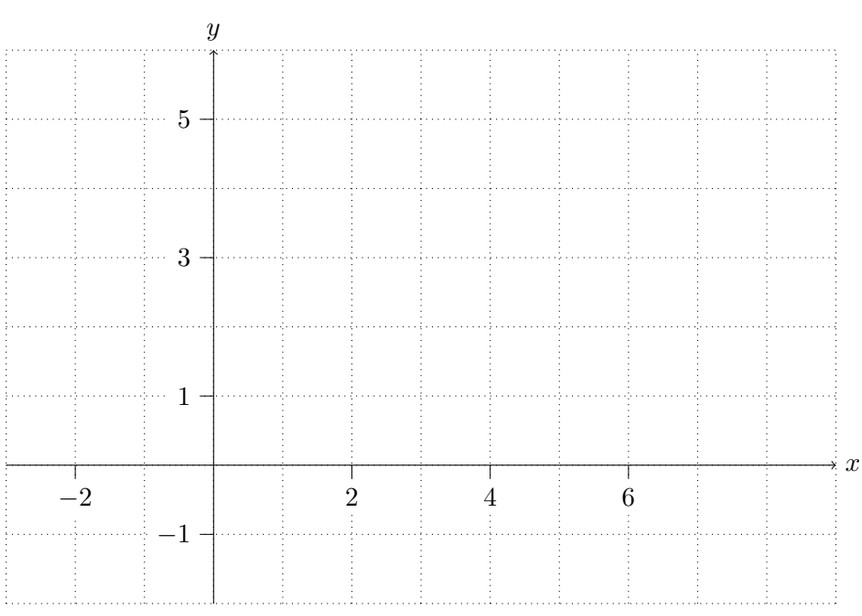
b. List open intervals where f is concave up.

c. List all x-coordinates of inflection points of f.

d. Give all x-coordinates where f has a relative maximum.

Score: /6

Problem 4: Sketch the graph of a single function g satisfying all of the following conditions. • The function g is continuous for all real numbers; • g is differentiable everywhere except at $x = 6$.; • $g(-1) = 5$; • $g'(-1) = 0 = g'(4)$; • $g'(x) > 0$ on open intervals $(-\infty, -1) \cup (6, \infty)$; • $g'(x) < 0$ if $-1 < x < 4$ or $4 < x < 6$; • $\lim_{x\to 6^-} g'(x) = -\infty$ and $\lim_{x\to 6^+} g'(x) = \infty$; • $g''(x) < 0$ on open intervals $(-\infty, 2) \cup (4, 6) \cup (6, \infty)$, and • $g''(x) > 0$ on the open interval $(2, 4)$.



Score: /5

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Calculators allowed in this part.

Problem 5: The association between velocity during exercise and blood lactate concentration after submaximal 800 m exercise of thoroughbred racehorses on sand and grass tracks has been studied. The lactate-velocity relationship can be described by the functions:

$$\ell_s(v) = 0.08e^{0.33v}, \quad \text{and} \quad \ell_g(v) = -0.87v^2 + 28.17v - 211.41,$$

where $\ell_s(v)$ and $\ell_g(v)$ are the lactate concentrations (in mmol/L) and v is the velocity (in m/s) of the horse during workout on sand and grass tracks, respectively.

a. Sketch the graph of both functions on the same window for v in the interval $[13, 17]$.

b. Specify the interval for v in $[13, 17]$ where running on a grass track yields more blood lactate in horses after they run 800 m.

c. On grass tracks, at what velocity should the horse run to produce the largest blood lactate concentration?

d. Find and interpret $\ell_s'(14)$. Include units.

e. Is ℓ_s an increasing function for any velocity? State your reasons.

f. Is ℓ_g an increasing function for any velocity? State your reasons.

Score: /8

Problem 6: The cost function to produce q delux-electric wire barbecue brushes is given by $C(q) = -8.9q^2 + 203q$. The demand equation is given by $p = -q^2 - 4q + 278$, where p is the price in dollars of each brush. Surgeon general warning: wire brush bristles found in barbecue may cause injuries.

a. Simplify the revenue function $R(q) = qp$ in terms of q only.

b. Find the marginal revenue. Include units.

c. The profit function is $P(q) = R(q) - C(q)$. Find the marginal average profit.

d. From the profit function, find the price that produces the maximum profit. What is the maximum profit.

e. Find the point of diminishing returns for the profit function.

Score: /8