Math 108
Fall 2017
Dr. Lily Yen

Test 1

Show all your work

Name: ____________________

Score: ___/43

No Calculator allowed in this part.

Problem 1: Determine the following limits analytically showing all steps. Use the symbols DNE, ∞, and $-\infty$ where appropriate.

a. $\displaystyle\lim_{x\to 9} \frac{9-x}{-\sqrt{x}+3} =$ **6**

$$\lim_{x\to 9} \frac{9-x}{-\sqrt{x}+3} = \lim_{x\to 9} \frac{9-x}{3-\sqrt{x}} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9} \frac{(9-x)(3+\sqrt{x})}{9-x}$$
$$= \lim_{x\to 9} 3+\sqrt{x} = 6$$

Score: /2

b. $\displaystyle\lim_{a\to -5^-} \frac{a^2-3a-40}{|a+5|} =$ **13**

$$\lim_{a\to -5^-} \frac{a^2-3a-40}{|a+5|} = \lim_{a\to -5^-} \frac{(a+5)(a-8)}{-(a+5)} = \lim_{a\to -5^-} -(a-8) = 13$$

Score: /2

c. $\displaystyle\lim_{y\to -\infty} \frac{100+70y-2y^3}{30-7y+5y^2} =$ ∞

$$\lim_{y\to -\infty} \frac{100+70y-2y^3}{30-7y+5y^2} = \lim_{y\to -\infty} \frac{y^3(\frac{100}{y^3}+\frac{70}{y^2}-2)}{y^2(\frac{30}{y^2}-\frac{7}{y}+5)} = \lim_{y\to -\infty} \frac{y(\frac{100}{y^3}+\frac{70}{y^2}-2)}{(\frac{30}{y^2}-\frac{7}{y}+5)}$$
$$= \lim_{y\to -\infty} \frac{y(-2)}{5} = \infty$$

Score: /2

Problem 2: Use the definition of continuity of a function at a point to find the value of the constant k that makes the function f below continuous.

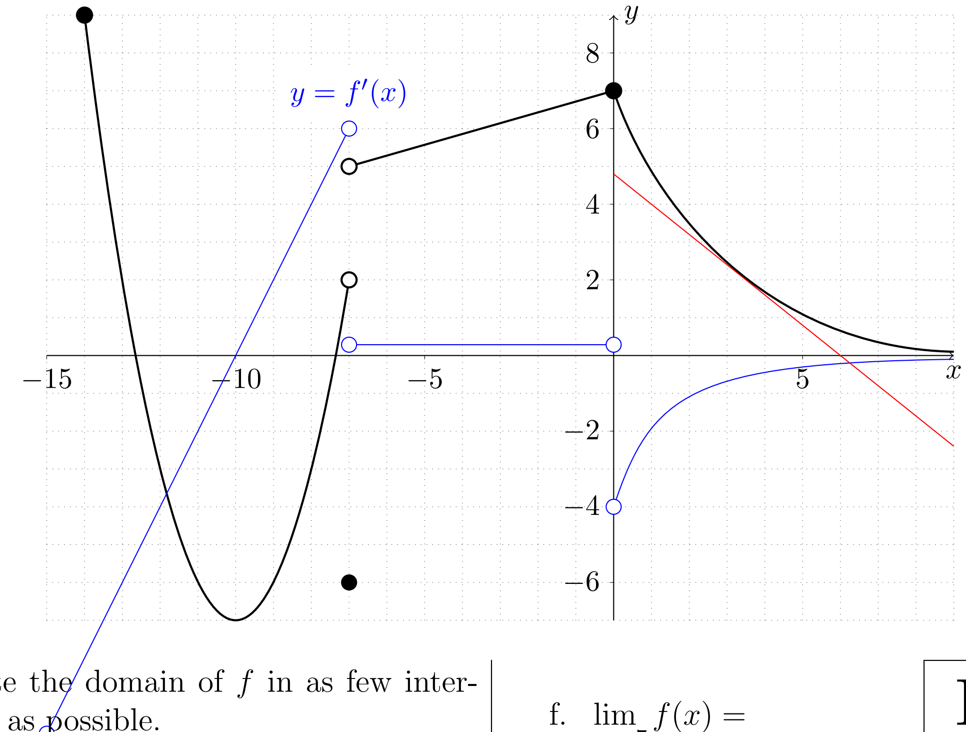
$$f(x) = \begin{cases} 2x^2 - k, & x \le -1 \\ k + x^3, & \text{otherwise.} \end{cases}$$

Each branch of f is a polynomial, so continuous. Therefore the only question is whether f is continuous at $x = -1$. Now, $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 2x^2 - k = 2 - k$, and $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} k + x^3 = k - 1$, and $f(-1) = 2 - k$, so continuity requires that $2 - k = k - 1$, so $3 = 2k$, so

$$k = \frac{3}{2}.$$

Score: /4

Problem 3: The graph of $y = f(x)$ is shown. Use the graph to answer the questions. Use the symbols DNE, ∞, and $-\infty$ where appropriate.



a. State the domain of f in as few intervals as possible.

$[-14, \infty)$

b. State where f is continuous in as few intervals as possible.

$[-14, -7) \cup (-7, \infty)$

c. $f(-7) =$ -6

d. $\lim_{x \to -7^-} f(x) =$ 2

e. $\lim_{x \to -7^+} f(x) =$ 5

f. $\lim_{x \to -7} f(x) =$ DNE

g. $\lim_{x \to -\pi} \dfrac{f(x) - f(-\pi)}{x + \pi} =$ $2/7$

h. $\lim_{h \to 0} \dfrac{f(h) - f(0)}{h} =$ DNE

i. $f'(-10) =$ 0

j. Draw the tangent line to f at $x = 3$ to estimate the value of $f'(3)$

-0.8

k. In the same grid above, **graph** $y = f'(x)$ for the domain of f.

Score: /12

Problem 4: Given $f(x) = \sqrt{2 - x}$, use the definition of the derivative to find $f'(x)$ and the equation of the tangent line at $x = -2$.

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2-x-h} - \sqrt{2-x}}{h} \\
&= \lim_{h\to 0} \frac{(\sqrt{2-x-h} - \sqrt{2-x})(\sqrt{2-x-h} + \sqrt{2-x})}{h(\sqrt{2-x-h} + \sqrt{2-x})} \\
&= \lim_{h\to 0} \frac{(2-x-h) - (2-x)}{h(\sqrt{2-x-h} + \sqrt{2-x})} = \lim_{h\to 0} \frac{-h}{h(\sqrt{2-x-h} + \sqrt{2-x})} \\
&= \lim_{h\to 0} \frac{-1}{\sqrt{2-x-h} + \sqrt{2-x}} = \frac{-1}{2\sqrt{2-x}}
\end{aligned}
$$

Therefore $f'(-2) = -\frac{1}{4}$. Since $f(-2) = 2$, the tangent line is given by

$$y - 2 = -\frac{1}{4}(x + 2) \qquad \text{or} \qquad x + 4y - 6 = 0.$$

Score: /4

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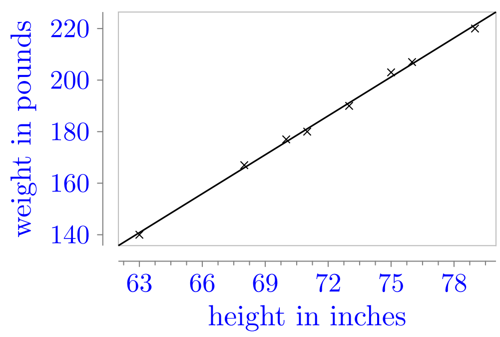
Calculators allowed from here on.

Problem 5: The table gives the average weight in pounds of North American men in their sixties for height in inches.

height (inches)	63	68	70	71	73	75	76	79
weight (pounds)	140	167	177	180	190	203	207	220

a. Examine and **draw** a scatter plot and determine the best **linear** model to fit the data, expressing weight as a function of height. State your model correct to 3 decimal places. Be sure to define your variables.



If x represents height and y represents weight, then $y = 5.035x - 176.364$

Score: /3

b. Use your model to predict on average the weight of a man in his sixties with a height of 7 feet, and comment on the accuracy of this prediction. Give 2 decimal place accuracy. Hint: A foot has 12 inches.

If $x = 7 \times 12 = 84$, then $y(84) = 246.55$ pounds.
Since 84 inches is outside the data, and since the line fits the data so well, this prediction is likely quite accurate despite of being an extrapolation (though likely not to two decimal places).

Score: /1

c. According to your model, on average, what would be the height of a man in his sixties with a weight of 130 pounds? Give 2 decimal place accuracy.

If $130 = 5.035x - 176.364$, then $x = 60.85$ inches.
This prediction extrapolates beyond the data which is always doubtful.

Score: /2

d. **State** and **interpret** the slope of the linear model including appropriate units for the slope using non-mathematical terms.

The slope is 5.035 pounds/inch, so for every inch gained in height, you would expect to gain 5.035 pounds in weight.

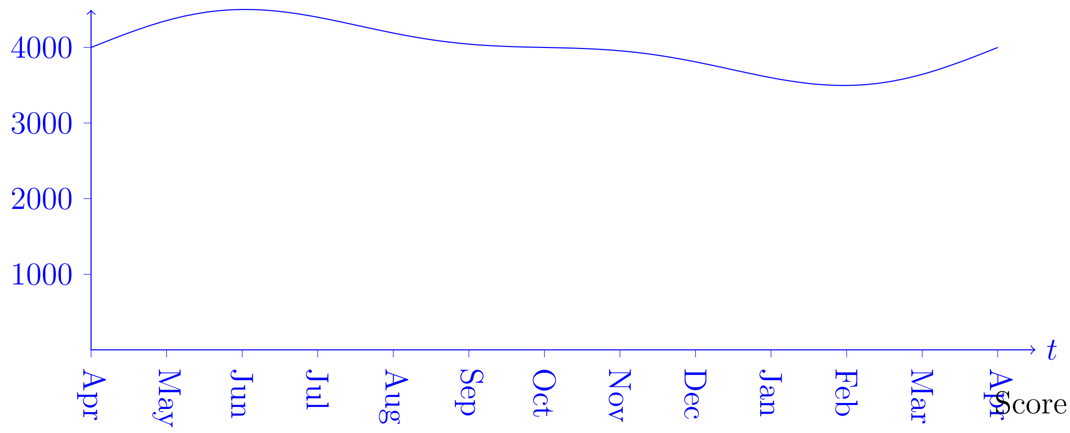
Score: /1

Problem 6: The population of a herd of deer in Waterton Lakes National Park is modelled by

$$P(t) = 4000 + 400\sin\left(\frac{\pi t}{6}\right) + 180\sin\left(\frac{\pi t}{3}\right)$$

where t is measured in months from the first of April.

a. Sketch a graph showing how this population varies with time for at least one year.



Score: /2

b. When is the herd largest? How many deer are in it at that time?

The maximum occurs when $t = 2.040$ (so in early June), and then the population numbers $P(2.040) = 4503$.

Score: /2

c. **State** and **interpret** $P(4)$ in non-mathematical terms. Include correct units.

$P(4) = 4191$. This is the size of the deer population in August.

Score: /1

d. Find the **average** rate of change in population over the first four months.

$\frac{P(4)-P(0)}{4-0} = 47.63\,\text{deer/month}$

Score: /1

e. Estimate the **instantaneous** rate of change for the population of this herd on the first of August. Use the chart below to document your estimate from average rate of change (ARC) to its limiting value. Give 4 decimal place accuracy.

interval	ARC	interval	ARC

Please list the functions you input into your graphing calculator.

$$Y_1 = P(t), \quad Y_2 = \frac{Y_1(x) - Y_1(4)}{x - 4}$$

Score: /3

Also check $Y_1'(4) = -198.9675$ using nDeriv on TI.

f. **State** and **interpret** $P'(4)$ in non-mathematical terms. Include correct units.

$P'(4) = -199.0\,\text{deer/month}$, so in early August the deer population shrinks almost 200 deer per month.

Score: /1