Math 108
Fall 2016
Dr. Lily Yen

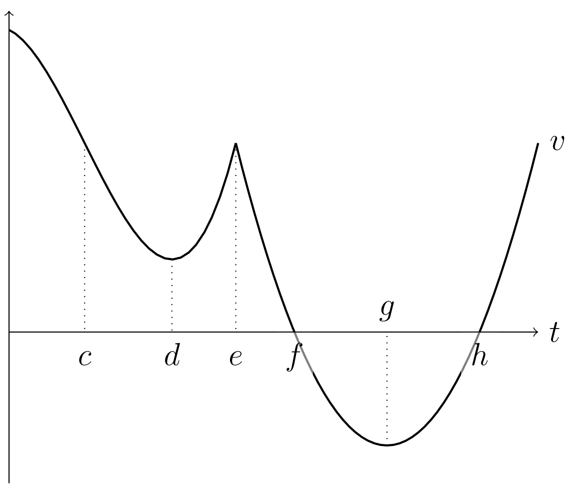
Test 3

Show all your work

Name: ____________
Number: ____________
Signature: ____________
Score: ___/40

No Calculator allowed in this part.

Problem 1: Recall that the derivative of the position function is velocity; and the derivative of the velocity function is acceleration. Given below is the graph of the velocity, $v(t)$ of a moving particle defined for $t \geq 0$. Assume that the particle moves along a straight line back and forth with $t = 0$ being the starting position. Answer the following questions.



a. List time intervals when the particle is moving closer to the starting position.

b. List open intervals where the particle has a positive acceleration.

c. List all times t where the acceleration of the particle changes sign.

d. Give all times t where the position function of the particle attains a relative extremum. Specify each as a relative maximum or a relative minimum.

e. Sketch the position function directly on the given coordinate system.

Score: /6

Problem 2: Sketch the graph of a single function satisfying all of the following conditions.
• The function is continuous and differentiable for all real numbers except at $x = 3$ where it has a vertical asymptote; • $f'(x) < 0$ everywhere it is defined; • A horizontal asymptote at $y = 1$; • $f''(x) < 0$ on open intervals $(-\infty, 3) \cup (4, 7)$, and • $f''(x) > 0$ on the open interval $(3, 4) \cup (7, \infty)$. Asymptotes must be labelled in your graph.

Score: /5

Test 3

Show all your work

Name: ____________________

Number: ____________________

Calculators allowed for this part.

Problem 3: Consider the following implicitly defined relationship:

$$y^2(x^2 + y^2) = 20x^2$$

a. Determine dy/dx.

Score: /3

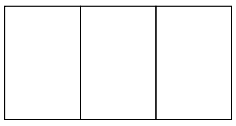
b. Find the equation of the tangent line at the point $(1, 2)$.

Score: /3

Problem 4: Find the absolute maximum and minimum of $f(x) = 2x - 3x^{2/3}$ on the interval $[-1, 2]$. You must use techniques of calculus to analyze the function.

Score: /5

Problem 5: An ecologist is conducting a research project on breeding Transylvanian dragons in captivity. She first must construct suitable pens. She wants a rectangular area with two additional fences across its width, as shown in the diagram. Find the maximum area she can enclose with 3600 m of fencing. Calculus techniques must be used.



Score: /5

Problem 6: Lake Capilano polluted by bacteria is treated with an antibacterial chemical. After t days, the number N of bacteria per milliliter of water is approximated by

$$N(t) = 20\left(\frac{t}{12} - \ln\left(\frac{t}{12}\right)\right) + 30$$

for t in $[1, 15]$.

a. When during this time will the number of bacteria be a minimum?

b. When during this time will the number of bacteria be a maximum? What is the maximum number?

Score: /5

Problem 7: A 10 ft ladder is placed against a building. The base of the ladder is slipping away from the building at a rate of 2 ft/min. Find the rate at which the top of the ladder is sliding down the building at the instant when the bottom of the ladder is 5 ft from the base of the building.

Score: /4

Problem 8: Sociologists have found that crime rates are influenced by temperature. In a prairie town of 100 000 people, the crime rate has been approximated as

$$C(T) = \frac{1}{10}(T - 60)^2 + 100$$

where C is the number of crimes per month and T is the average monthly temperature in degrees Fahrenheit. The average temperature for May was $55°$, and by the end of May, the temperature was rising at the rate of $5°$ per month. How fast is the crime rate rising at the end of May?

Score: /4