Math 105-01
Fall 2019
Dr. Lily Yen

Quiz 6

Show all your work

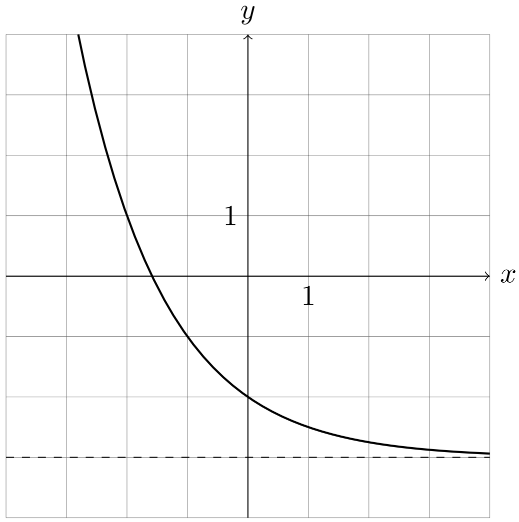
Family name: ______________

Given name: ______________

Student number: ______________

Score: ___/16

Problem 1: Determine what sequence of transformations were applied on the graph of $y = 2^x$ to produce the graph below. State explicitly the function of the graph.



Down 3, reflect in y-axis: $y = 2^{-x} - 3$

Score: /2

Problem 2: Determine whether each quantity below represents a linear or exponential growth.

a. The value of a certain household item depreciates by \$100 per year.

$\boxed{\text{Linear}}$

b. A school of fish is losing one-fourth of its population every two years.

$\boxed{\text{Exponential}}$

c. The concentration of a drug decreases be 5 % every ten minutes.

$\boxed{\text{Exponential}}$

Score: /3

Problem 3: A jar with a volume of $1000\,\text{cm}^3$ contains bacteria that double in number every minute. If the jar is full in 60 minutes, how long will it take for the jar to be one-eighth full?

59 minutes, the jar is half full (since the number doubles every minute). 58 minutes, the jar is a quarter full. 57 minutes, the jar is an eighth full.

Score: /2

$\boxed{\quad /7}$

Problem 4: Find the domain of $f(x) = \ln\sqrt{x+5} - \ln(x+1)$.

The domain of $\sqrt{x}$ is $[0,\infty)$, and the domain of $\ln(x)$ is $(0,\infty)$.
Hence, for the sake of the square root, $x \geq -5$; for the sake of $\ln\sqrt{x+5}$, you need that $x > -5$; and for the sake of $\ln(x+1)$, you need $x > -1$.
Therefore that domain of f is $(-1,\infty)$.

Score: /2

Problem 5: At what annual rate of return, compounded continuously, would your investment double in six years?

Let P be the initial investment. If $2P = Pe^{6r}$, then $6r = \ln 2$, so

$$r = \frac{\ln 2}{6} \approx 0.1155 \approx 11.55\,\%$$

Score: /2

Problem 6: In a lake, one-fourth of the water is replaced by clean water every year. A spill of $16\,000\,\mathrm{m}^3$ of toxic chemicals takes place in the lake. Let $T(n)$ denote the amount of toxin left in the lake after n years.

a. Find a formula for $T(n)$.

Every year one-fourth of the toxins are replaced by clean water, so three-fourths remain. Hence

$$T(n) = 16\,000\left(\frac{3}{4}\right)^n$$

b. How much toxin will be left after 12 years?

$$T(12) = 16\,000\left(\frac{3}{4}\right)^{12} \approx 506.8\,\mathrm{m}^3$$

c. When will 80 % of the toxins be eliminated?

If 80 % have been eliminated, then 20 % remain:
$0.20 \cdot 16\,000 = 16\,000\,(3/4)^n$, so
$0.20 = (3/4)^n$, so
$\ln(0.20) = \ln\left((3/4)^n\right) = n\ln(3/4)$, so

$$n = \frac{\ln(0.2)}{\ln(3/4)} \approx 5.59 \text{ years}$$

Score: /5

/9