Stat 101
Summer 2023 Session 1
Dr. Lily Yen

Activity 5-1

Show all your work

Name: ________________

Number: ________________

Signature: ________________

Score: ___/12

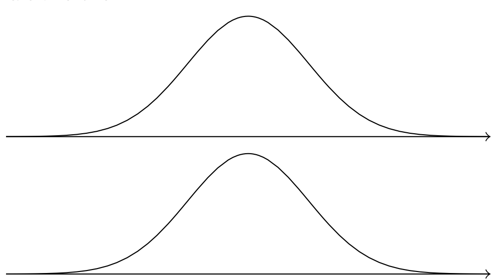
State all Excel functions used.

Problem 1: In Vancouver's BMO Marathon in May, friends, Matt and Sarah both completed the 10-km fast walk. Matt joined Men's ages 30 to 34 group while Sarah, in Women's ages 25 to 29 group. Matt finished his walk in $1:22:28 = 4948$ seconds while Sarah, in $1:31:53 = 5513$ seconds. According to the finishing statistics of their respective group, answer the following questions.

- Mean finishing time of Men's ages 30 to 34 group is 4313 seconds, with $\sigma = 583$ seconds.
- Mean finishing time of Women's ages 25 to 29 group is 5261 seconds with $\sigma = 807$ seconds.
- The distributions of finishing times for both groups are approximately Normal.

Remember that faster (less) time means better performance in racing.

a. Draw these two normal distributions marking the corresponding mean and standard deviation.



b. Find the Z-scores for both and mark on the corresponding normal curves.

Matt's Z-score

$$z = \frac{4948 - 4313}{583} \approx 1.089193825$$

Sarah's

$$z = \frac{5513 - 5261}{807} \approx 0.312267658$$

c. What percentile did Matt place in his group?

Since Matt's Z-score is approximately 1.089193825, to find its percentile, or left tail, one uses normal cdf NORM.DIST(1.089193825,0,1,TRUE) to get 0.8619657521. This means 86% of participating men in Matt's group finished in a shorter time than Matt, or Matt placed on the 14th percentile.

d. What percentile did Sarah place in her group?

Similarly for Sarah, one uses $NORM.DIST(0.312267658, 0, 1, TRUE)$ to get 0.6225813767, meaning that 62% of participants finished walking faster than Sarah. Therefore, Sarah placed on the 38th percentile.

Score: /6

/6

Problem 2: The National Vaccine Information Centre estimates that 90% of Canadians have had chickenpox by the time they reach adulthood.

a. Suppose we take a random sample of 100 Canadian adults. Is the use of binomial distribution appropriate for calculating the probability that exactly 97 out of 100 randomly sampled Canadian adults had chickenpox during childhood?

b. Find the probability that at most 15 out of 100 randomly sampled Canadian adults *have not* had chickenpox.

c. Find the probability that at least half of 10 randomly sampled Canadian adults had chickenpox during childhood.

a. If a random sample of 100 Canadian adults is taken, the sample size is large, however the use of binomial distribution is still appropriate for calculating the probability that exactly 97 out of 100 randomly sampled Canadian adults had chickenpox. In fact, it is $\binom{100}{97}0.9^{97}0.1^3 = BINOM.DIST(97, 100, 0.9, 0) \approx 0.005892$.

b. The probability that at most 15 out of 100 randomly sampled Canadian adults have NOT had chickenpox during childhood is approximated by a normal distribution with $\mu = 100 \times 0.1 = 10$ and $\sigma = \sqrt{100 \times 0.9 \times 0.1} = 3$.
$NORM.DIST(15, 10, 3, TRUE) \approx 0.033159$

c. The probability that at least half of 10 randomly sampled Canadian adults had chickenpox during childhood is 1− the sum of 0, 1, 2, 3, and 4 adults out of 10 sampled with chickenpox during childhood. Cumulative binomial distribution gives us this sum.
$$1 - BINOM.DIST(4, 10, 0.9, 1) \approx 0.9998531$$

Score: /3

Problem 3: Sickle cell anemia is a genetic blood disorder where red blood cells lose their flexibility and assume an abnormal, rigid, "sickle" shape, which results in a risk of various complications. If both parents are carriers of the disease, then a child has

- a 1/4 chance of having the disease,
- a 1/2 chance of being a carrier, and
- a 1/4 chance of neither having the disease nor being a carrier.

If both parents who are carriers of the disease have 3 children, find each of the following probabilities.

a. The probability that none of the 3 children will have the disease.

b. The probability that at least one will neither have the disease nor be a carrier.

c. The probability that exactly 2 are carriers.

a. The probability that none of the 3 children will have the disease. $\binom{3}{0}(3/4)^3(1/4)^0 = \frac{27}{64}$

b. The probability that at least one will neither have the disease nor be a carrier.
$1 - \frac{27}{64} = \frac{37}{64}$

c. The probability that exactly 2 are carriers. $\binom{3}{2}(1/2)^2(1/2)^1 = \frac{3}{8}$.

Score: /3

/6