Math 101
Summer 2018
Dr. Lily Yen

Test 3

Show all your work

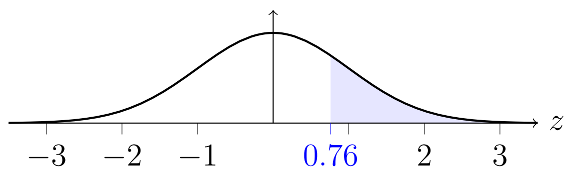
Name: ________________
Number: ________________
Signature: ________________
Score: ___/43

A TI-83/84 calculator allowed.

Problem 1: Evaluate each of the following probabilities from a standard normal probability distribution using either the tables or your graphing calculator. In either case, label the relevant value(s) on the z-axis and shade corresponding area. State calculator command and entries if you use the calculator. Round to 4 decimal places.

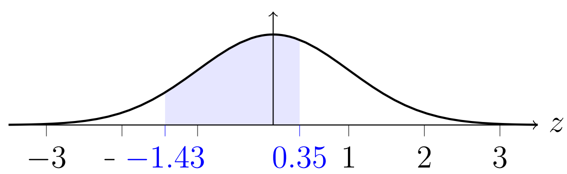
a. $P(Z > 0.76) = \boxed{0.2236}$

normalcdf$(0.76, 1 \times 10^{99})$



b. $P(-1.43 \le Z < 0.35) = \boxed{0.5605}$

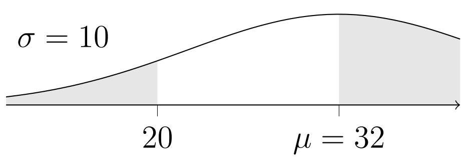
normalcdf$(-1.43, 0.35)$



Score: /4

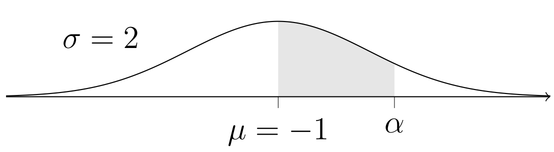
Problem 2: For the normal probability distributions with their corresponding mean and standard deviation, find the value of the variable indicated in each. State calculator command and entries for each part. Round to 4 decimal places.

a. Find the area of the shaded region. $\boxed{0.6151}$



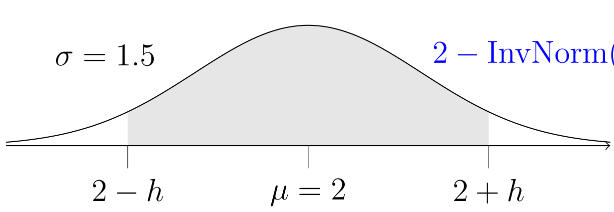
normalcdf$(-10^{99}, 20, 32, 10) + \frac{1}{2}$

b. Find α if the shaded region has area 0.4000. $\boxed{1.5631}$



InvNorm$(0.5 + 0.4, -1, 2)$

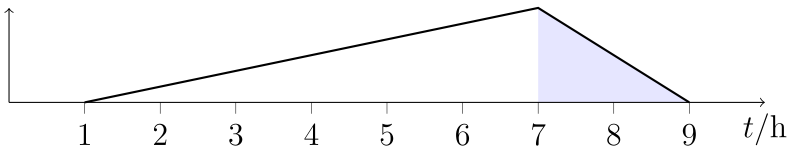
c. Find h if the shaded region has area 0.8888. $\boxed{2.3892}$



$2 -$ InvNorm$(\frac{1-0.8888}{2}, 2, 1.5)$

Score: /6

Problem 3: Suppose that the distribution of the hours of sleep per night for university students during final exam period is given by the graph shown. What proportion of university students get at least 7 hours of sleep per night during the finals?



The area of the triangle has to be 1. Since the base is 8, the height is $\frac{1}{4}$. The area of the shaded region in the figure is thus $\frac{2\times 1/4}{2} = \frac{1}{4}$.
Hence one quarter of the students get at least seven hours of sleep, that is,
$P(X \geq 7) = 1/4$.

Score: /4

Problem 4: The distribution of IQ scores in Burnaby's Gifted Program is a nonstandard normal distribution with a mean of 120 and a standard deviation of 13. Answer the following.

a. Find the first quartile Q_1, which is the IQ score separating the bottom 25% from the top 75%.

$\text{InvNorm}(0.25, 120, 13) \approx 111$

b. What percentage of students in the Gifted Program has an IQ greater than 135?

$\text{normalcdf}(134.5, 10^{99}, 120, 13) \approx 13.2\,\%$

c. Find the probability that a randomly selected student from the program has an IQ less than 100.

$\text{normalcdf}(0, 100.5, 120, 13) \approx 6.68\,\%$

d. Eighty percent of the students in the program has an IQ between 120 plus or minus how many IQ points?

$\text{InvNorm}(\frac{1}{2} + 0.80/2, 120, 13) - 120 \approx 16.7$

e. Find the IQ score separating the top 1% from the others.

$\text{InvNorm}(0.99, 120, 13) \approx 150$

f. In a group of 25 students in Parkcrest Elementary's Gifted Program, what is the probability that the average IQ is between 120 and 135?

$\text{normalcdf}(119.5, 135.5, 120, 13/\sqrt{25}) \approx 57.6\,\%$

Score: /12

Problem 5: A simple random sample of 50 students (including males and females) from Capilano University is obtained, and each person's red blood cell count (in cells per microliter) is measured. The sample mean is 4.78. The population standard deviation for red blood cell count is 0.49.

a. Find the best point estimate of the mean red blood cell count of Cap's students.

   $\bar{x} = 4.78$

b. Construct a 95% confidence interval estimate of the mean red blood cell count of our students.

   Here $\alpha = 0.05$ and
   $$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 1.96\frac{0.49}{\sqrt{50}} = 0.136$$
   The confidence interval, $\bar{x} - E < \mu < \bar{x} + E$, is therefore $4.64 < \mu < 4.91581857478364$

c. The normal range of red blood cell counts for adults is 4.7 to 6.1 for males and 4.3 and 5.4 for females. What does the confidence interval suggest about these normal ranges?

   The confidence interval is more or less within normal range, but these students do not seem athletic.

Score: /5

Problem 6: The length of a featured movie is known to follow a normal distribution. Listed below are 12 lengths (in minutes) of randomly selected movies.

110 96 125 94 132 120 136 154 149 94 119 132

a. Construct a 99% confidence interval estimate of the mean length of all movies.

   Here $\bar{x} = 122$, $s = 20.4$, $\alpha = 0.01$, and you need a Student's t-distribution with $n - 1 = 11$ degrees of freedom, so
   $$E = t_{\alpha/2}\frac{s}{\sqrt{n}} = 3.11\frac{20.4}{\sqrt{12}} = 18.3$$
   The confidence interval, $\bar{x} - E < \mu < \bar{x} + E$, is therefore $103 < \mu < 140$

b. Assuming that it takes 30 minutes to empty a theatre after a movie, clean it, allow time for the next audience to enter, and show previews, what is the minimum time that a theatre manager should plan between start times of movies, assuming that this time will be sufficient for typical movies?

   Using the upper end of the confidence interval above, $30 + 140 = 170$ minutes between movies would be sufficient 99.5 % of the time.

Score: /4

Problem 7: The trend of thinner Miss America winners has generated charges that the contest encourages unhealthy diet habits among young women. Listed below are body mass indices (BMI) for recent Miss America winners. Use a 0.01 significance level to test the claim that recent Miss America winners are from a population with a mean BMI less than 20.16, which was the BMI for winners from the 1920s and 1930s. Do recent winners appear to be significantly different from those in the 1920s and 1930s?

19.5, 20.3, 19.6, 20.2, 17.8, 17.9, 19.1, 18.8, 17.6, 16.8

a. Define your symbols.

Let $\bar{x}$ be the sample mean; s, sample deviation; n, sample size; μ, population mean of body mass index; H_0, null hypothesis; H_1, alternative hypothesis; and df is degree of freedom in a t statistic; α is the level of significance.

b. State the null hypothesis.

$H_0 : \mu = 20.16$ or $\mu \geq 20.16$

c. State the alternative hypothesis.

$H_1 : \mu < 20.16$, also the claim.

d. State the test statistic and compute.

Enter data into a list in the graphing calculator and use 1-Var Stat to calculate sample mean and sample standard deviation.

$$\bar{x} = 18.76, \quad s \approx 1.1862, \quad n = 10, \quad df = 9, \quad \alpha = 0.01.$$

$$t = \frac{18.76 - 20.16}{1.1862/\sqrt{10}} \approx -3.7322$$

e. Draw the critical value on an appropriate graph and locate your test statistic.

$\mathrm{invT}(0.01, 9) \approx -2.8214$. Since we are doing the left-tail rejection region, our $t = -3.7322 < -2.8214$ the critical value. Thus the statistic falls in the rejection region.

f. State your conclusion using everyday language. No technical terms.

There is sufficient evidence at the significance level of 0.01 to support the claim that the average BMI of recent Miss America winners is lower than 20.16.

Score: /8