Math 101-02
Fall 2015
Dr. Lily Yen

Midterm 3

Show all your work

Name: ____________________

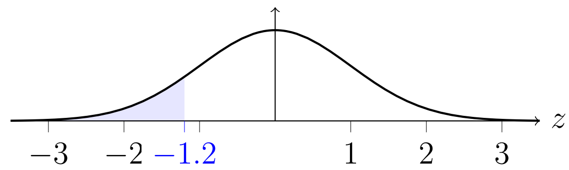
Score: ___/42

A TI-83/84 calculator allowed.

Problem 1: Evaluate each of the following probabilities from a standard normal probability distribution using either the tables or your graphing calculator. In either case, label the relevant value(s) on the z-axis and shade corresponding area. State calculator command and entries if you use the calculator. Round to 4 decimal places.

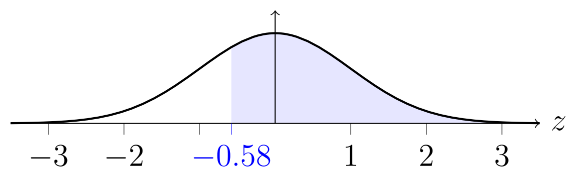
a. $P(Z < -1.2) =$ **0.1151**

normalcdf$(-1 \times 10^{99}, -1.2)$



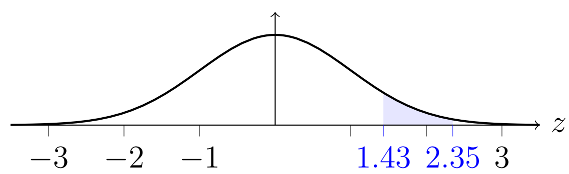
b. $P(Z \geq -0.58) =$ **0.7190**

normalcdf$(-0.58, 1 \times 10^{99})$



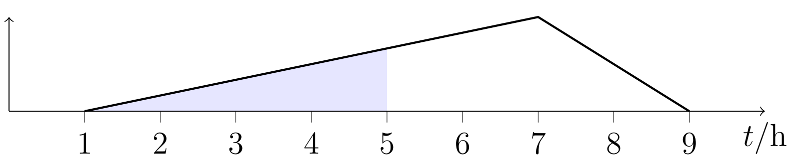
c. $P(1.43 \leq Z < 2.35) =$ **0.0670**

normalcdf$(1.43, 2.35)$



Score: /6

Problem 2: Suppose that the distribution of the hours of sleep per night for university students during final exam period is given by the graph shown. What proportion of university students get at most 5 hours of sleep per night during the finals?



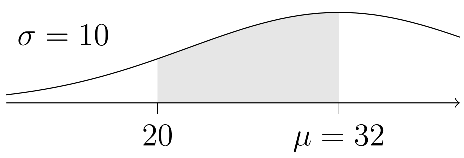
The area of the triangle has to be 1. Since the base is 8, the height is $\frac{1}{4}$. The slope of the left side of the triangle is therefore $\frac{1/4}{6} = \frac{1}{24}$. At $x = 5$, the height is then $\frac{1}{24} \times 4 = \frac{1}{6}$. The area of the shaded region in the figure is thus $\frac{4 \times 1/6}{2} = \frac{1}{3}$.
Hence one third of the students get at most five hours of sleep.

Score: /3

Problem 3: For the normal probability distributions with their corresponding mean and standard deviation, find the value of the variable indicated in each. State calculator command and entries for each part. Round to 4 decimal places.

a. Find the area of the shaded region.

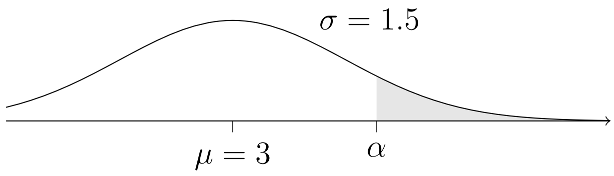
0.3849



normalcdf$(20, 32, 32, 10)$

b. Find α if the shaded region has area 0.1020.

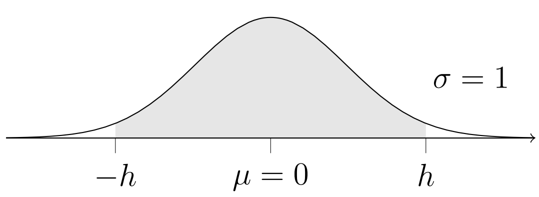
4.9054



InvNorm$(1 - 0.1020, 3, 1.5)$

c. Find h if the shaded region has area 0.9600.

2.0537



$-$ InvNorm$(\frac{1-0.9600}{2}, 0, 1)$

Score: /6

Problem 4: Find the third quartile of the standard normal distribution to 4-decimal place accuracy.

InvNorm$(3/4) = 0.6745$.

Score: /2

Problem 5: The number of hours a student is on the internet each day is normally distributed with a mean of 5.6 hours and a standard deviation of 1.8 hours.

a. What percentage of students uses the internet for less than 5.6 hours each day?

50%

b. What is the probability that a randomly selected student is on the internet more than 10 hours each day?

normalcdf$(10, 1 \times 10^{99}, 5.6, 1.8) = 0.007\,254$

c. Ninety percent of the students uses the internet for less than how many hours?

InvNorm$(0.90, 5.6, 1.8) = 7.9\,\text{h}$

d. Eighty percent of the students uses the internet between 5.6 plus or minus how many hours?

So 10 % are below the bottom of the range. Now InvNorm$(0.10, 5.6, 1.8) = 3.3\,\text{h}$, so 80 % of the students are within $5.6\,\text{h} - 3.3\,\text{h} = 2.3\,\text{h}$ of the mean.

e. For our class of 26 students, what is the probability that their average internet usage per day is between 5 and 6 hours?

normalcdf$(5, 6, 5.6, 1.8/\sqrt{26}) = 0.8268$.

Score: /7

Problem 6: Regarding Canadian airport security, in a random sample of 405 workers with access to restricted areas, 10 had criminal ties.

a. Use the sample results to find a 95 % confidence interval for the proportion of workers with access to restricted areas of Canadian airports who have criminal ties. Present the results of your CI calculation in complete sentence form with specific reference to this application.

The endpoints of the confidence interval are given by $p \pm z\sqrt{\frac{1}{n}p(1-p)}$, where $z = \text{InvNorm}(0.975) = 1.960$ for a 95 % interval.
In the present case, $n = 405$ and $p = \frac{10}{405} = \frac{2}{81}$, so the CI is

$$(0.009\,578, 0.039\,80).$$

If you were to test samples and calculate confidence intervals as above many times, the true proportion of workers in restricted areas with criminal ties would be captured by 95 % of the CI's.

b. Suppose there are 113 000 workers with clearance to airport restricted areas in Canada, up to how many workers with criminal ties have access to restricted areas?

Using the CI above, one would expect between 1082 and 4498 such workers have criminal ties.

Score: /5

Problem 7: The weekly weight of garbage generated per household is known to follow a normal distribution. A random sample of 10 households found these weights in kilograms:

2.1 5.8 3.9 4.2 3.1 7.8 6.4 4.9 4.7 5.1

Construct a 90 % confidence interval estimate of the average weekly weight of garbage per household.

Here $n = 10$ is small and σ is unknown, so use Student's t-distribution with $n - 1 = 9$ degrees of freedom. The endpoints of the CI for μ are then $\bar{x} \pm t_{\alpha/2,9}\frac{s}{\sqrt{n}}$.
In the present case, $\alpha = 0.10$, so the `invT` program on your graphing calculator yields that $t_{\alpha/2,9} = t_{0.05,9} = -1.833$. Moreover, $\bar{x} = \frac{1}{10}\sum_i x_i = 4.80\,\text{kg}$ and $s^2 = \frac{1}{9}\sum_i (x_i - \bar{x})^2$ so $s = 1.63\,\text{kg}$. (Of course you can also find these values by typing the data into your calculator and letting it do the rest of the work.) Hence the CI for μ is

$$(3.85\,\text{kg}, 5.75\,\text{kg}).$$

Score: /5

Problem 8: In a CBC poll during Cycle to Work Week, it was found that 2 % of Richmond residents cycles to work. How large a random sample is needed to estimate the percentage of Richmond residents that cycles to work if the estimate is to be accurate within 2 % at the 90 % confidence level?

The endpoints of the confidence interval are $p \pm z\sqrt{\frac{1}{n}p(1-p)} = 2\,\% \pm 2\,\%$, where $p = 2\,\%$ and $z = \text{InvNorm}(0.95) = 1.64$ for a 90 % CI. Thus $z\sqrt{\frac{1}{n} \times 0.02 \times 0.98} = 0.02$, so $z^2\frac{1}{n} \times 0.0196 = 0.02^2$, so $n = \frac{0.0196z^2}{0.02^2} = 133$. Thus the sample needs 133 people.

Score: /3

Problem 9: A veggie sticks manufacturer weighed 90 bags of veggie sticks where each bag had an advertised weight of 80 grams. The weights of the sample had a mean of 81.4 grams and a standard deviation of 2.3 grams.

a. Based on this sample, construct a 98 % CI for the mean weight of all bags of veggie sticks of the above type.

   Since $n = 90$ is large, you may assume that the weights follow a normal distribution. Since σ is unknown, use Student's t-distribution with $n - 1 = 89$ degrees of freedom. The endpoints for the CI for μ are then $\bar{x} \pm t_{\alpha/2,89}\frac{s}{\sqrt{n}}$.
   In the present case, $\alpha = 0.02$, so the `invT` program on your graphing calculator yields that $t_{\alpha/2,89} = t_{0.01,89} = -2.369$. Hence the CI for μ is

   $$(80.83\,\text{g}, 81.97\,\text{g})$$

b. How large a random sample is needed to be 99.5 % confident that the sample mean weight is within 1 gram of the mean weight of all bags of veggie sticks with advertised weights of 80 grams? What assumption did you have to make to come up with this number?

   You know that the standard deviation of the sample is 2.3 g. Assume that standard deviation of the population is 2.3 g.
   Since the error, $E = z\frac{\sigma}{\sqrt{n}}$, it follows that $n = (\frac{z\sigma}{E})^2$. In the present case, $E = 1$ and $z = \text{InvNorm}(0.0025) = -2.807$, so $n = 41.68$. Thus you need a sample of at least 42 bags.

Score: /5