Math 101-02
Fall 2015
Dr. Lily Yen

Midterm 2

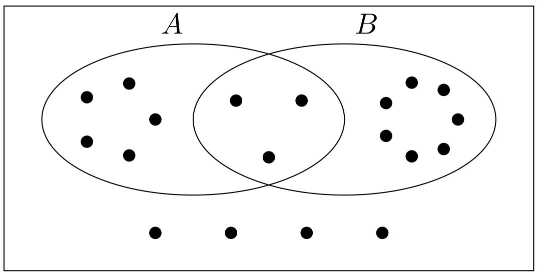
Show all your work

Name: ____________________

Score: ___/50

A TI-83/84 calculator allowed.

Problem 1: Each dot in the Venn diagram represents an equally likely event in the sample space S. Suppose one of them is randomly selected. Find each probability below.



a. $P(\bar{A}) = \boxed{\frac{11}{19} \approx 57.9\,\%}$

b. $P(A \text{ or } B) = \boxed{\frac{15}{19} \approx 78.9\,\%}$

c. $P(B|A) = \boxed{\frac{3}{8} \approx 37.5\,\%}$

d. $P(A|\bar{B}) = \boxed{\frac{5}{9} \approx 55.6\,\%}$

e. $P(A \text{ and } B) = \boxed{\frac{3}{19} \approx 15.8\,\%}$

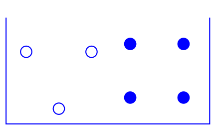
f. Are A and B independent? Explain.

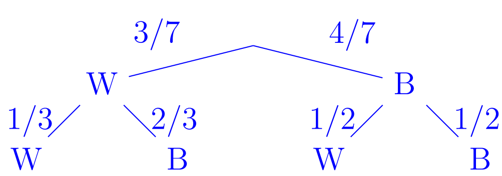
$P(B) = \frac{10}{19} \neq \frac{3}{8} = P(B|A)$, so A and B are NOT INDEPENDENT.

Score: /6

Problem 2: Draw a pot containing 3 white balls and 4 black balls. Randomly select two balls from the pot without replacement.

a. Draw a probability tree for the scenario.





b. Find the probability that exactly one ball is black.

$P(WB) + P(BW) = \frac{3}{7} \times \frac{2}{3} + \frac{4}{7} \times \frac{1}{2} = \frac{2}{7} + \frac{2}{7} = \frac{4}{7} \approx 57.1\,\%$

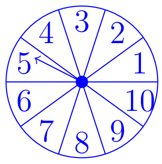
c. Find the probability that both balls are the same colour.

$P(WW) + P(BB) = \frac{3}{7} \times \frac{1}{3} + \frac{4}{7} \times \frac{1}{2} = \frac{1}{7} + \frac{2}{7} = \frac{3}{7} \approx 42.9\,\%$

Score: /6

Problem 3: Draw a spinner with ten evenly spaced regions numbered distinctly from 1 to 10.

a. Find the probability that all three spins come up at 3.



$(\frac{1}{10})^3 = \frac{1}{1000} = 0.100\,\%$

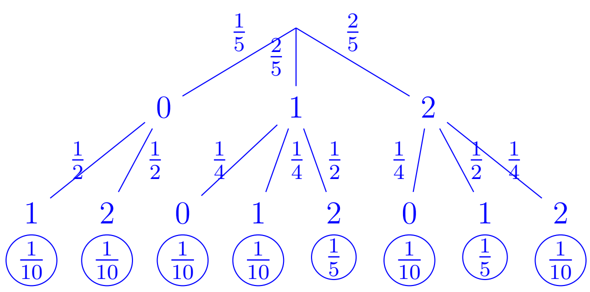
b. Find the probability that each of the three spins comes up at a different number.

The chance that the first spin come up with any number is of course 1. The chance that the second spin comes up with a different number is then $\frac{9}{10}$. The chance that the third spin is different from both of the first two is $\frac{8}{10} = \frac{4}{5}$. The probability that all three are different is thus $1 \times \frac{9}{10} \times 45 = \frac{18}{25} = 72\,\%$.

Score: /3

Problem 4: A box contains a \$0, two \$1, and two \$2 bills. Randomly select 2 bills from the box without replacement. Let the Random Variable be X, the total dollar value of the 2 bills.

a. Find the probability distribution of X and display it as a table.



x	\$1	\$2	\$3	\$4
$P(X = x)$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{10}$

b. Find the probability that the 2 bills selected are worth at least \$3. Use proper notation.

$P(X \geq 3) = P(X = 3) + P(X = 4) = \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$.

Score: /6

Problem 5: The table below gives the probability distribution for the number of passengers per vehicle arriving at Capilano University.

No. of passengers	0	1	2	3	4
Probability	0.35	0.25	0.20	0.15	0.05

a. What is the probability that a vehicle has at least one passenger? Use proper notation.

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.35 = 0.65.$

b. Find the expected number of passengers per vehicle.

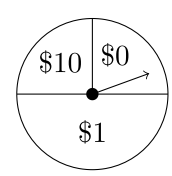
$E(X) = 0 \cdot P(X = 0) + 1 \cdot P(X = 1) + 2 \cdot P(X = 2) + 3 \cdot P(X = 3) + 4 \cdot P(X = 4) = 0 \times 0.35 + 1 \times 0.25 + 2 \times 0.20 + 3 \times 0.15 + 4 \times 0.05 = 1.30$

c. Find the standard deviation of the number of passengers per vehicle.

$E(X^2) = 0^2 \cdot P(X = 0) + 1^2 \cdot P(X = 1) + 2^2 \cdot P(X = 2) + 3^2 \cdot P(X = 3) + 4^2 \cdot P(X = 4) = 0 \times 0.35 + 1 \times 0.25 + 4 \times 0.20 + 9 \times 0.15 + 16 \times 0.05 = 3.20$, so the standard deviation is $\sqrt{E(X^2) - E(X)^2} = \sqrt{3.20 - 1.30^2} = 1.23$.

Score: /6

Problem 6: Consider the spinner shown. Identify each variable described as Binomial (B) or Not Binomial (N):



a. the number of spins of the pointer until a $10 comes up. **N**

b. the "amount won" on a single spin of the pointer. **N**

c. the total amount won on 5 spins of the pointer. **N**

d. the number of times a $10 comes up in 5 spins. **B**

e. the number of times a $0 comes up before the first $10 comes up. **N**

Score: /5

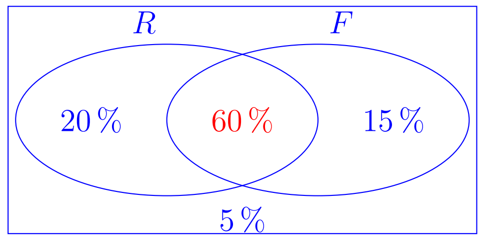
Problem 7: In Casino Royale, if you bet on the number 7 in roulette, you have a 1/38 chance of winning $175, otherwise you lose $5. Is this game in your favour? Show your work to back up your claim.

Your expected winnings are $\$175 \times \frac{1}{38} - \$5 \times (1 - \frac{1}{38}) = \$-0.26$, so this game is NOT IN YOUR FAVOUR.

Score: /3

Problem 8: In Capilano University's Summer Camp program, both Robotics (R) and Film making (F) are available. Some of the campers did both activities while 20 % did only robotics, 15 % did only Film making, and 5 % did neither. Answer the following using a Venn diagram or probability formulas. Give your answers as %.

a. Find the % that did both.



b. For this example, what is $P(R \text{ or } F)$? Explain in plain English what this percentage of campers did.

$P(R \text{ or } F) = 1 - 5\,\% = 95\,\%$ of the campers did robotics, film making, or both.

c. Given that a particular camper did Robotics, what is the probability that the camper also did Film making?

$P(F|R) = \frac{P(F \text{ and } R)}{P(R)} = \frac{60\,\%}{80\,\%} = \frac{3}{4} = 75\,\%.$

Score: /7

Problem 9: Thirty percent of 2 million voters in BC favour the NDP. In a random sample of 50 BC voters, finding the the probability of getting exactly 8 people who favour the NDP is the calculation of a particular probability of binomial random variable.

a. Using the usual binomial notation, determine the values of x, n, p, and q.

$$x = 8, \quad n = 50, \quad p = 30\,\% = 0.30, \quad q = 1 - p = 0.70.$$

b. Find the probability of getting exactly 8 people out of 50 BC voters who favour the NDP.

$P(X = 8) = \text{binompdf}(50, 0.3, 8) \approx 1.10\,\%.$

c. Find the probability of getting 5 or fewer votes for NDP out of 50 BC voters.

$P(X \leq 5) = \text{binomcdf}(50, 0.3, 5) \approx 0.0723\,\%.$

Score: /4

Problem 10: Assume that male and female births are equally likely, and that the birth of any child does not affect the probability of the gender of any other children. In a family of 4 children, find each of the following probability.

a. At least one girl.

If the family DOES NOT have at least one girl, then they have four boys. The likelihood of that is $(\frac{1}{2})^4 = \frac{1}{16}$.
Therefore the likelihood of at least one girl is $1 - \frac{1}{16} = \frac{15}{16} = 93.8\,\%$.

b. At most two boys.

The chance of at most two boys is $P(0) + P(1) + P(2)$ or $1 - P(3) - P(4)$. The latter is slightly easier to calculate: $1 - 4 \times \frac{1}{16} - \frac{1}{16} = \frac{11}{16} = 68.8\,\%$.

Score: /4