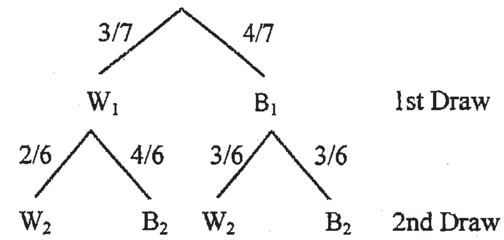
Section 4.4 Problems
+ 4.5

1. a) 10/13 b) 3/13 c) 1/4 d) 3/7 e) 12/13 f) 4/13

2. a) $P(W_1 \text{ and } W_2) = \frac{3}{7}\cdot\frac{2}{6} = \frac{6}{42} = \frac{1}{7}$



b) $P([B_1 \text{ and } W_2] \text{ or } [W_1 \text{ and } B_2])$
$= \frac{4}{7}\cdot\frac{3}{6} + \frac{3}{7}\cdot\frac{4}{6} = \frac{24}{42} = \frac{4}{7}$

c) $1 - P(B_1 \text{ and } B_2) = 1 - \frac{4}{7}\cdot\frac{3}{6} = \frac{30}{42} = \frac{5}{7}$

3. Let W = {pointer ends in white region}; S = {pointer ends in shaded region}

a) $P([S_1 \text{ and } W_2] \text{ or } [W_1 \text{ and } S_2]) = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2}$

b) $P([0 \text{ and } 10] \text{ or } [10 \text{ and } 0] \text{ or } [5 \text{ and } 5]) = \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{4} = \frac{5}{16}$

4. a) i) $P([0 \text{ and } 20] \text{ or } [20 \text{ and } 0] \text{ or } [10 \text{ and } 10]) = \frac{1}{4}\cdot\frac{1}{3} + \frac{1}{4}\cdot\frac{1}{3} + \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{3}$

ii) $P(\overline{0} \text{ and } 0) = \frac{3}{4}\cdot\frac{1}{3} = \frac{1}{4}$

b) i) $P([0 \text{ and } 20] \text{ or } [20 \text{ and } 0] \text{ or } [10 \text{ and } 10]) = \frac{1}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} + \frac{2}{4}\cdot\frac{2}{4} = \frac{6}{16} = \frac{3}{8}$

ii) $P([\overline{0} \text{ and } 0] \text{ or } [0 \text{ and } 0]) = \frac{3}{4}\cdot\frac{1}{4} + \frac{1}{4}\cdot\frac{1}{4} = \frac{4}{16} = \frac{1}{4}$

5. a) $P(C_1 \text{ and } C_2) = \frac{13}{52}\cdot\frac{12}{51} = \frac{156}{2652} = \frac{1}{17}$

b) $P(K_1 \text{ and } K_2) = \frac{4}{52}\cdot\frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$

6. a) $P(6 \text{ and } 6) = \frac{1}{6}\cdot\frac{1}{6} = \frac{1}{36}$

   b) $P(\overline{5} \text{ and } \overline{5}) = \frac{5}{6}\cdot\frac{5}{6} = \frac{25}{36}$

7. Let L = {Lisa solves problem}; I = {Ian solves problem}

   a) P(L and I) = P(L)·P(I) since L and I are independent!
   = (0.90)(0.70) = 0.63

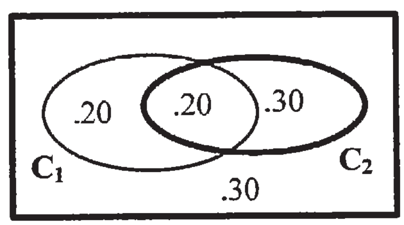
   b) P(L or I) = P(L) + P(I) – P(L and I)
   = 0.90 + 0.70 – 0.63
   =0.97

8. Let H = {heavy smoker}; E = {having emphysema}

   P(H) = 200/1000 = 1/5 = 0.2; P(H|E) = 13/20 = 0.65

   Since P(H) ≠ P(H|E), the events "being a heavy smoker" and "having emphysema" are dependent.

9. Let C_1 = {getting first contract}; C_2 = {getting second contract}



   $P(C_1 \text{ and } C_2) = P(C_1)\cdot P(C_2)$ since C_1, C_2 are independent
   = (0.40)(0.50) = 0.20

   $P([C_1 \text{ and } \overline{C_2}] \text{ or } [\overline{C_1} \text{ and } C_2]) = 0.20 + 0.30 = 0.50$

10. $P(B_1 \text{ and } B_2 \text{ and } B_3) = \frac{6}{11}\cdot\frac{5}{10}\cdot\frac{4}{9} = \frac{4}{33} \approx 0.1212$

11. a) $P(B_1 \text{ and } B_2 \text{ and } B_3 \text{ and } B_4 \text{ and } B_5) = \frac{6}{11}\cdot\frac{5}{10}\cdot\frac{4}{9}\cdot\frac{3}{8}\cdot\frac{2}{7} = \frac{1}{77} \approx 0.0130$

   b) $P(\text{at least one white}) = 1 - P(\text{all black}) = 1 - \frac{1}{77} = \frac{76}{77} \approx 0.9870$

12. a) $P(C_1 \text{ and } C_2 \text{ and } C_3 \text{ and } C_4 \text{ and } C_5) = \frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}\cdot\frac{10}{49}\cdot\frac{9}{48} \approx 0.0004952$

b) $P(\text{at least one club}) = 1 - P(\text{no clubs}) = 1 - P(\overline{C}_1 \text{ and } \overline{C}_2 \text{ and } \overline{C}_3 \text{and } \overline{C}_4 \text{and } \overline{C}_5)$

$$= 1 - \frac{39}{52}\cdot\frac{38}{51}\cdot\frac{37}{50}\cdot\frac{36}{49}\cdot\frac{35}{48} \approx 0.7785$$

13. a) $P(\overline{0}_1 \text{ and } \overline{0}_2 \text{ and} \ldots \text{and } \overline{0}_{10}) = P(\overline{0}_1)\cdot P(\overline{0}_2)\ldots P(\overline{0}_{10}) = \frac{1}{2}\cdot\frac{1}{2}\cdot \;\ldots\; \cdot\frac{1}{2} = \frac{1}{2^{10}}$

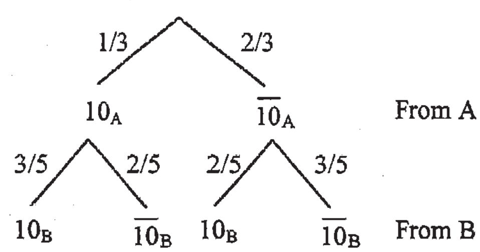
$$= \frac{1}{1024} \approx 0.0009766$$

b) $P(\text{at least one } 0) = 1 - P(\text{no } 0\text{'s}) = 1 - \frac{1}{1024} = \frac{1023}{1024} \approx 0.9990$

14. $P([0_A \text{ and } 20_B] \text{ or } ([20_A \text{ and } 0_B] \text{ or } ([10_A \text{ and } 10_B]) = \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{2}{4} = \frac{4}{12} = \frac{1}{3}$

15. $P(10_B) = P([10_A \text{ and } 10_B] \text{ or } ([\overline{10}_A \text{ and } 10_B]$

$$= \frac{1}{3}\cdot\frac{3}{5} + \frac{2}{3}\cdot\frac{2}{5} = \frac{7}{15}$$



16. $P(20_A|10_B) = \frac{P(20_A \text{ and } 10_B)}{P(10_B)} = \frac{\frac{1}{3}\cdot\frac{2}{5}}{\frac{7}{15}} = \frac{2}{7}$

17. $P(\text{some prize at least once}) = 1 - P(\text{no prize every week}) = 1 - \left(\frac{27}{28}\right)^{52} \approx 0.8491$

18. a) i) $P(\text{Rh+ and } \overline{B}) = 28/47$
ii) $P(\text{A or Rh-}) = 22/47$

b) $P(O|\text{Rh-}) = 6/12 = 1/2$

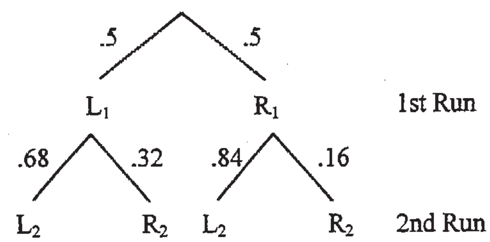
c) i) $P([\text{Rh+ and Rh-}] \text{ or } [\text{Rh- and Rh+}]) = \frac{35}{47}\cdot\frac{12}{46}+\frac{12}{47}\cdot\frac{35}{46} \approx 0.3885$

ii) P([A and A] or [B and B] or [AB and AB] or [O and O])

$$= \frac{13}{47}\cdot\frac{12}{46}+\frac{9}{47}\cdot\frac{8}{46}+\frac{6}{47}\cdot\frac{5}{46}+\frac{19}{47}\cdot\frac{18}{46} \approx 0.2775$$

d) $P(\text{at least one type A}) = 1 - P(\text{none has type A}) = 1 - \frac{34}{47}\cdot\frac{33}{46}\cdot\frac{32}{45}\cdot\frac{31}{44}\cdot\frac{30}{43} \approx 0.8186$

19. a) $P(L_1 \text{ and } L_2) = (.5)(.68) = 0.34$



b) $P(L_2) = P([L_1 \text{ and } L_2] \text{ or } [R_1 \text{ and } L_2])$
$= (.5)(.68) + (.5)(.84) = 0.76$

c) $P(L_1|L_2) = \frac{P(L_1 \text{ and } L_2)}{P(L_2)} = \frac{0.34}{0.76} \approx 0.4474$

20. Let S = {single parent}; E = {employed}

a) P(S or E) = P(S) + P(E) – P(S and E)
= 0.70 + 0.60 – P(S and E)
But P(S and E) = P(S)·P(E|S) = (0.70)(0.80) = 0.56
So
P(S or E) = 0.70 + 0.60 – 0.56 = 0.74

b) $P(S|E) = \frac{P(\text{S and E})}{P(E)} = \frac{0.56}{0.60} \approx 0.9333$

Therefore 93.3% of employed people are single parents.

21. a) i) P(morning) = 6/18 = 1/3
ii) P(afternoon or Tues) = 13/18
iii) Wednesday, because P(Wed) = 6/18 = 1/3 and P(Wed|morning) = 2/6 = 1/3
Since P(Wed) and P(Wed|morning) are both equal to 1/3, it follows that "picking morning" and "picking Wed" are independent

b) P(at least one picks Wed) = 1 – P(neither picks Wed)
= 1 – (12/18)(11/17) = 29/51 ≈ 0.5686

22. a) $P(0 \text{ and } 0 \text{ and } 0) = \frac{15}{35} \cdot \frac{14}{34} \cdot \frac{13}{33} = \frac{13}{187} \approx 0.0695$

b) P([0 and 10 and 20] or [0 and 20 and 10] or [10 and 0 and 20] or [10 and 20 and 0] or [20 and 0 and 10] or [20 and 10 and 0])

$$= \frac{15 \cdot 12 \cdot 8 + 15 \cdot 8 \cdot 12 + 12 \cdot 15 \cdot 8 + 12 \cdot 8 \cdot 15 + 8 \cdot 15 \cdot 12 + 8 \cdot 12 \cdot 15}{1440 \cdot 1439 \cdot 1438} \approx 0.2200$$

23. a) $P(7 \text{ and } 7 \text{ and } 7) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{1000} = 0.001$

b) P([1 and a different no. and another different no.] or [2 and a different no. and another different no.] or …)

$$= \underbrace{\frac{1}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} + \frac{1}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} + \cdots + \frac{1}{10} \cdot \frac{9}{10} \cdot \frac{8}{10}}_{\text{10 terms}} = \frac{720}{1000} = 0.72$$

OR

$$= \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} = \frac{720}{1000} = 0.72$$

First spin any number

Second spin any number except the one that occurred on first spin

Third spin any number except the two that occurred on first & second spins