Problems for You to Do: (Sections 5.4 – 5.5)

1. Suppose that pregnancy lengths are normally distributed with a mean of 268 days and a standard deviation of 9 days.
   a) What is the probability that a pregnancy lasts at least 260 days?
   b) 80% of the pregnancies last longer than how many days?
   c) What is the probability that a baby is born sometime during the 268th day after conception? (Assume that the normal is centred at the start of the 268th day.)
   d) If a woman has 4 children and if the lengths of her pregnancies are independent of each other, what is the probability that
      i) the average (mean) length of her 4 pregnancies is less than 265 days?
      ii) at least half of her pregnancies were greater than 270 days?
   e) 15 different mothers gave birth to babies in one day at Vancouver General Hospital. What is the probability that the average length of the 15 pregnancies was at least 270 days?
   f) What is the probability that the average length of 10 pregnancies randomly selected is at least 270 days?

2. If the population of Vancouver teenagers (12 – 17 years) watches an average of 14 hours of TV per week with a standard deviation of 3 hours, what is the probability that the average viewing time of a random sample of 64 teenagers is within 15 minutes of the population average?

3. If you take a very large random sample from some population, is it true that the population will always be approximately normally distributed? Briefly explain why or why not.

4. True or False?
   The Central Limit Theorem says that if you take a large random sample from any population than the distribution of the *sample* will be approximately normal.

5. In a study of job training at a small business college, the times required to learn how to use a word processor are found to be normally distributed with a mean of 460 minutes and a standard deviation of 75 minutes.
   a) What is the probability that one trainee selected at random will learn the word processor in less than 10 hours?
   b) If a local company routinely hires the best 25% of trainees (based on speed in learning the processor), what is the cut-off learning time for this group?
   c) If 32 people are randomly selected, find the probability that their average training time is between 420 and 450 minutes.

6. Assume that when you pay for 60 minutes of time, the meter time that you actually get from a randomly selected meter is normally distributed with a mean of 62 minutes and a standard deviation of 2 minutes. (The city has set the meters to be"generous" on average.)
   a) What is the probability that a randomly selected meter will give a time less than 60 minutes?
   b) What is the probability that a random sample of 25 meters will produce an average meter time less than 60 minutes?

SUN, SATURDAY, NOVEMBER 6, 1999

Time flies when you're at a Vancouver meter

Annoyed and suspicious of the four parking tickets she had received, lawyer Caterine Sas decided to conduct a study of parking meters in Vancouver. Her survey results revealed that many of them were inaccurate, confirming her mistrust of the meters.

Ms. Sas hired her clerk, Melissa Hayward, to conduct a random survey of 25 parking meters in the downtown core. Only five were accurate. While nine meters ran fast, cheating parkers of about two minutes and 40 seconds, five meters ran slow, giving a reprieve of three minutes and 40 seconds. Six of the meters appeared to have a mind of their own, running slow on some days and fast on others.

7. Human birth weights for full term babies are normally distributed with a mean of 3.4 kg and a standard deviation of 0.5 kg.
   a) What is the proportion of birth weights that are at least 3.0 kg?
   b) What is the probability that a random sample of 4 babies will produce an average birth weight of at least 3.0 kg?
   c) Why can the Central Limit Theorem be used in Part b) even though the sample size is less than 30?
   d) What birth weight interval would be considered unusually low for the average birth weight of a random sample of 50 babies?

THE VANCOUVER SUN, FRIDAY, MARCH 17, 2000

8. **Life creeps in the slow lane**

Vancouver drivers now spend an average of 70 minutes per day commuting by car.

Suppose that there are 200,000 Vancouver drivers who commute daily by car, and that the standard deviation of their commute times is 30 minutes.
   a) Above what size of random sample should you use the finite population correction factor in the calculation of the standard error?
   b) For a random sample of 500 Vancouver drivers who commute daily by car, what is the probability that the sample mean commute time is between 69 and 71 minutes? (Assume the population mean is as given in the headline above.)

9. If you take a large random sample from an extremely large population, which of the following statements are always true?
   a) The distribution of the sample is approximately normal.
   b) The distribution of the sample mean is approximately normal.
   c) The distribution of the population mean is approximately normal.

10. Suppose the Lottery Corporation introduces a new game "So You Want to Win $100" with the following prize structure.

So You Want to Win $100

Prize	$100	$50	$0
Probability	1/1000	1/200	994/1000

Mean: $0.35
SD: $4.73

If you play this game 1000 times what is the probability that you win a *total* of less than $200? Hint: think mean!

11. The diameters of the base of all trees harvested by a logging company were found to be normally distributed with a mean of 52 cm and a standard deviation of 7 cm.
    a) If you sample a log at random, what is the probability that its diameter will be within 4 cm of the average diameter of all logs?
    b) A log is considered to be superior if the base diameter is over 65 cm. What percent of logs harvested will be considered superior?
    c) If you randomly sample 8 logs, find the probability that their sample mean will be within 2 cm of the population mean.
    d) How large a random sample would you have to take to be 95% certain that the average diameter of the sampled logs is within 1.5 cm of the population average diameter?

12. Suppose that the waist circumferences of 18 – 24 year old males are normally distributed with a mean of 33 inches and a standard deviation of 2 inches.
    a) The thinnest 15% of 18 – 24 year old males have a waist circumference of less than how many inches?
    b) If one male is randomly selected from the 18 – 24 age group, what is the probability that his waist circumference is at least 34 inches?
    c) If a random sample of 16 males is selected from the 18 – 24 age group, what is the probability that the mean waist circumference of this sample is at least 36 inches? (10 decimal places)

13. Suppose lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.
    a) What is the probability that a randomly selected sample of 25 pregnant females has a mean pregnancy length within 20 days of 268 days?
    b) There is a 20% probability that a random sample of 25 pregnancies has a mean length exceeding what value?

14. For the "Straight and Scramble" version of the Daily 3 you calculate for the winnings a
    - mean of \$0.49
    - standard deviation of \$9.58

    for each one dollar played.

DAILY 3

DAILY 3 is a B.C.-only lottery featuring three ways to play—Straight, Scramble and Straight & Scramble. Playing DAILY 3 is as easy as 1, 2, 3!

PRICE:

- \$1, \$2, \$5 or \$10

DRAWS HELD:

- daily
- you can buy tickets up to 9 p.m. Pacific time for that day's draw

If you play this game 365 times, what is the probability that your average winnings for the 365 plays will be at most 70 cents?

15. If human birth weights for full term babies are normally distributed with a mean of 3.4 kg and a standard deviation of 0.5 kg
    a) find the birth weight exceeded by only the heaviest 2% of the babies.
    b) find the probability that a random sample of 225 babies produces a mean birth weight between 3.3 and 3.5 kg.

16. Thanksgiving turkeys have weights that are normally distributed with a mean of 6.5 kg and a standard deviation of 2.5 kg.

    a) What percentage of the turkeys weighs at least 5 kg? (2 decimals)
    b) What percentage of the turkeys weighs between 6 and 7 kg?
    c) How many of the turkeys weigh no more than 10 kg?
    d) The lightest 10% of the turkeys weigh at most how many kg? (2 decimals)
    e) The turkeys are shipped to the supermarkets in boxes containing 25 turkeys each.
        i) What is the probability that the average weight of the contents of a box is between 6 and 7 kg? (4 decimal accuracy)
        ii) What percentage of the boxes have a *total* weight of their contents exceeding 200 kg? Hint: think mean!

17. Suppose that the weights of Grade A large eggs are normally distributed with a mean of 50 grams and a standard deviation of 4 grams.
    a) What proportion of Grade A large eggs weighs at most 52 grams?
    b) What is the probability that the average weight of a dozen eggs is at most 52 grams?
    c) What weight interval would be considered unusually high for the average weight of a random sample of 25 eggs?

18. In general terms briefly explain what is meant by the *sampling distribution* of the mean.

19. For women aged 18 – 24, systolic blood pressures are normally distributed with a mean of 114.8 and a standard deviation of 13.1.
    a) If 12 women are selected randomly, find the probability that their mean systolic blood pressure is at least 120.
    b) Given that the above sample size is only 12, explain why the Central Limit Theorem can be used.
    c) What is the probability that exactly half of the women in the sample of 12 had a mean systolic blood pressure of at least 120?
    d) What is the probability that at least 9 of the 12 women in the sample had a mean systolic blood pressure less than 120?

20. Weights of Grade A large eggs are normally distributed with a mean of 50 grams and a standard deviation of 4 grams.
    a) What percentage of egg cartons (containing a dozen eggs) has at least 4 eggs in them weighing more than 52 grams?
    b) What proportion of egg cartons (containing a dozen eggs) has average weights less than 49 grams?
    c) A supermarket receives a shipment of 400 cartons of eggs. What is the probability that less than 20% of the shipment has average carton weights less than 49 grams?

21. The population of weights of men has a mean of 173 pounds and a standard deviation of 30 pounds. An elevator has a maximum capacity of 32 men, and it is considered overloaded if the total weight exceeds 5952 pounds (i.e., if the mean weight of the 32 occupants exceeds 186 lbs).
    a) If 32 randomly selected men enter the elevator, what is the probability that the elevator is overloaded?

    Optional: (if Finite Population Correction Factor has been discussed)
    b) If 32 men are selected (without replacement) from a *finite* population, what is the smallest population for which the above calculation is still reasonably accurate?
    c) If the 32 men are selected from a population of size 300, what adjustment should be made to your calculation in part (a)?