Sections 8.1 – 8.5 Problems

1. a) $H_0: \mu = 11490$ km where μ = mean annual driving distance
   $H_1: \mu < 11490$ km. for women aged 16-24 years.

   b) $z = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$

   c) Critical region: $z < -z_{.05} = \text{invNorm}(.05, 0, 1) \approx -1.6449$

   d) $z = \dfrac{9750 - 11490}{4750 / \sqrt{750}} \approx -10.0320$

   e) Reject H_0 in favour of H_1 since –10.0320 is less than –1.6449.

   f) The sample provides strong evidence that the average annual driving distance for women aged 16-24 years is lower than the overall average for all women.

   g) (i) concluding that women aged 16-24 years drive less when, actually, they do not; charging these women a lower premium when they should be paying more; the insurance company loses money that it should be collecting.

   (ii) not rejecting the claim that younger women drive the same amount as others when they actually do drive less; younger women paying the "regular" premium when they should be paying less; the insurance company is "overcharging" younger women and making more money than it should.

2. a) a number calculated from the sample data that allows the researcher to decide between the two competing hypotheses; i.c. whether to reject the null hypothesis or not.

   b) the probability of making a Type I error; i.e., of rejecting the null hypothesis when it is actually true.

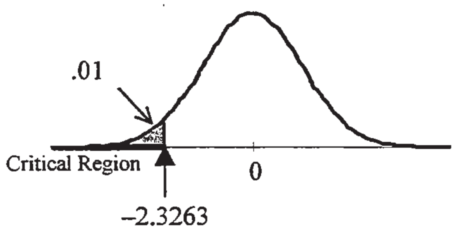
3. a) $H_0: \mu \geq 36$ where μ is the mean "stand-by" lifetime of the manufacturer's cell
   $H_1: \mu < 36$ phone batteries, in hours of continuous use.

   b) When the test statistic $z = \dfrac{\bar{x} - 36}{s / \sqrt{60}}$ is less than $-z_{.01} \approx -2.3263$
   (where $\bar{x}$ and s are the sample mean and SD, respectively)

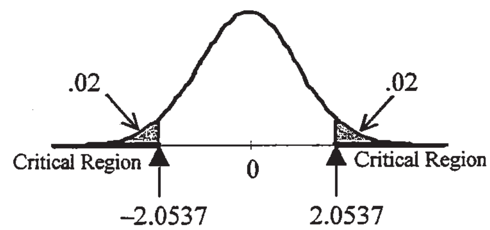
   (c) Test statistic $z = \dfrac{35.8 - 36}{11.2 / \sqrt{60}} \approx -1.2910$ which is greater than –2.3263 (NOT in critical region)
   Conclusion: Fail to reject H_0

   (d) Type II

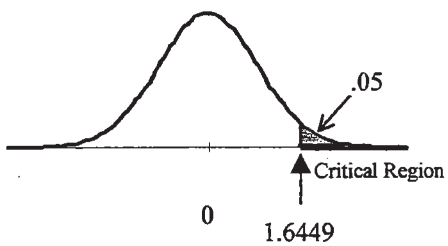
4. a) $-z_{.01} = \text{invNorm}(.01, 0, 1) \approx -2.3263$



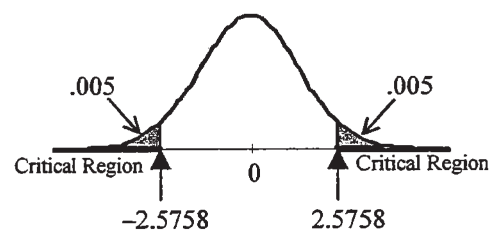
b) $z_{.02} = \text{invNorm}(.98, 0, 1) \approx 2.0537$



c) $z_{.05} = \text{invNorm}(.95, 0, 1) \approx 1.6449$



b) $z_{.005} = \text{invNorm}(.995, 0, 1) \approx 2.5758$



5. a) H_0: $\mu \geq 61$ min. where μ = average parking time for Vancouver parking meters.
H_1: $\mu < 61$ min.

b) When the test statistic $z = \dfrac{\bar{x} - 61}{s/\sqrt{n}}$ is less than $-z_{.10} \approx -1.2816$.
(where $\bar{x}$, s and n are the sample mean, SD and sample size, respectively)

c) Test statistic $z = \dfrac{(60.8 - 61)}{0.5/\sqrt{50}} \approx -2.8284$ which is less than -1.2816 (i.e., in the C.R.)
Conclusion: Reject H_0 in favour of H_1; i.e., there is strong evidence to indicate that the average parking time in Vancouver meters is less than 61 minutes, i.e., that the city engineers' claim is false.

6. a) H_0: $\mu = 46$ cm where μ = average seat width of basketball fans.
   H_1: $\mu > 46$ cm

   b) (i) Test Statistic $z = \dfrac{\bar{x} - 46}{s/\sqrt{n}} = \dfrac{48 - 46}{6/\sqrt{80}} \approx 2.9814$

   Critical region: $z > z_{.01} = \text{invNorm}(.99, 0, 1) \approx 2.3263$
   The test statistic value of 2.9814 is in the critical region.
   Conclusion: At a significance level of .01, reject H_0 in favour of H_1 ; i.e., the owners' study showed that the average seat width of basketball fans is larger than 46 cm.

   (ii) Critical region: $z > z_{.001} = \text{invNorm}(.999, 0, 1) \approx 3.0902$.
   Now, the test statistic value of 2.9814 is *not* in the critical region.
   Conclusion: At a significance level of .001, fail to reject H_0; i.e., the study did not provide strong enough evidence to conclude that the average seat width of basketball fans exceeds 46 cm.

7. The critical region is the set of values of the test statistic that would lead to the rejection of the null hypothesis.

8. a) H_0: $\mu \geq 5$ min. where μ = average reduction in 10km run times for "out of shape"
   H_1: $\mu < 5$ min. young adults that use the new physical fitness program.

   b) (i) Critical region: $z < -z_{.05} = \text{invNorm}(.05, 0, 1) \approx -1.6449$
   Test statistic $z = \dfrac{\bar{x} - 5}{s/\sqrt{n}} = \dfrac{(4.8 - 5)}{.5/\sqrt{60}} \approx -3.0984 < -1.6449$ (in C.R.)
   Conclusion: At the 0.05 level reject H_0 in favour of H_1 ; i.e., there is strong evidence to show that the average reduction in run times is less than 5 minutes; i.e., that the new fitness program's promoters' claim is false.

   (ii) Critical region: $t < -t_{.05}$ (11 degrees of freedom), or $t < -1.796$
   Test statistic $t = \dfrac{(4.8 - 5)}{.5/\sqrt{12}} \approx -1.3856$ (not in C.R.)
   Conclusion: At the 0.05 level fail to reject H_0 ; i.e., the sample data did not provide strong enough evidence against the promoters' claim.
   Additional assumption: the run time reductions are normally distributed.

9. a) ordinal

b) $H_0: p = .5$ where p = proportion of male students, aged 18-22 years,
$H_1: p > .5$ that evaluate their fitness level as OK or better.
(Note: "majority" means more than half.)

Test statistic: $z = \dfrac{\hat{p} - .5}{\sqrt{\dfrac{(.5)(.5)}{n}}}$

Critical region: $z > z_{.01} = \text{invNorm}(.99, 0, 1) \approx 2.3263$

Sample data: 26 out of 36 self-evaluate as VG, G or OK; $\hat{p} = \dfrac{26}{36}$

Evaluate test statistic: $z = \dfrac{\dfrac{26}{36} - .5}{\sqrt{\dfrac{(.5)(.5)}{36}}} \approx 2.6667 > 2.3263$ (in C.R.)

Conclusion: Reject H_0 in favour of H_1; i.e., the sample gives strong evidence that the majority of males aged 18-22 years evaluate their fitness levels as OK or better.

10. a) H_0: $\mu = 1$ mg/cig. where μ = mean level of tar inhaled by human smokers of Carlton cigarettes
H_1: $\mu > 1$ mg/cig.

Test statistic: $t = \dfrac{\bar{x} - 1}{s / \sqrt{n}}$

Critical region: $t > t_{.10}$ (7 degrees of freedom); i.e., $t > 1.415$

Evaluate test statistic: $t = \dfrac{(1.1625 - 1)}{.28253 / \sqrt{8}} \approx 1.6268 > 1.415$ (in C.R.)

Conclusion: Reject H_0 in favour of H_1; i.e., the sample provided strong evidence that the mean tar contact for human smokers is higher than the advertised amount.

b) The tar levels for all smokers are normally distributed

c) Type I error; i.e., rejecting the null hypothesis when it is actually true.

11. a) $\mu > 30$ mi/gal. where μ = mean mileage for Taurus cars with the new fuel injection design.

b) $\mu \le 30$ mi/gal.

c) H_0: $\mu \le 30$ mi/gal

d) H_1: $\mu > 30$ mi/gal.

e) Deciding that the mean mileage for the Taurus with the new fuel injection design is greater than 30 mi/gal. when in fact it is not.

f) Not rejecting the claim that the mean mileage for Taurus cars with the new fuel injection has not improved when in fact the mileage has improved.

12. H_0: $\mu = 420$ hr. where μ = mean time between failures for modified radios used in
H_1: $\mu > 420$ hr. light aircraft.

Test statistic: $z = \dfrac{\bar{x} - 420}{s/\sqrt{n}}$

Critical region: $z > z_{.05} \approx 1.6449$

Evaluate test statistic: $z = \dfrac{485 - 420}{24/\sqrt{35}} \approx 16.0227 > 1.6449$ (in C.R.)

Conclusion: Reject H_0 in favour of H_1; the sample provided strong evidence that the modified ratios had a longer mean time between failures.

13. H_0: $\mu \ge 10$ points where μ is the mean sensory rating of the population.
H_1: $\mu < 10$ points

Test statistic: $t = \dfrac{\bar{x} - 10}{s/\sqrt{n}}$

Critical region: $t < -t_{.01}$ (15 degrees of freedom), or $t < -2.602$

Evaluate test statistic: $t = \dfrac{8.33 - 10}{1.96/\sqrt{16}} \approx -3.4082 < -2.602$ (in C.R.)

Conclusion: Reject H_0 favour of H_1; i.e., this sample provided strong evidence that it came from a population with a mean VAS rating of less than 10.

14. a) $\bar{x} \approx 124.2313, s \approx 22.5225$

b) H_0: $\mu = 114.8$ where μ = mean systolic blood pressure reading (in mm Hg)
H_1: $\mu \neq 114.8$ for women aged 18-24 with a new strain of viral infection.

Test statistic: $t = \dfrac{\bar{x} - 114.8}{s/\sqrt{n}}$

Critical region: $t < -t_{.025}$ or $t > t_{.025}$ (15 degrees of freedom);
i.e., $t < -2.131$ or $t > 2.131$

Evaluate test statistic: $t = \dfrac{124.2313 - 114.8}{22.5225/\sqrt{16}} \approx 1.6750$ (Not in C.R.)

Decision: Fail to reject H_0

Conclusion: At the 0.05 level the sample did not provide strong evidence to suggest that the mean systolic blood pressure readings of women aged 18-24 with the new strain of viral infection is different from that of healthy women in that age group.

15. H_0: $p = 0.07$ where p is the proportion of no-shows under the new reservation system.
H_1: $p < 0.07$

Test statistic: $z = \dfrac{\hat{p} - 0.07}{\sqrt{\dfrac{(0.07)(0.93)}{n}}}$

Critical region: $z < -z_{.05} \approx -1.6449$

Evaluate test statistic: $z = \dfrac{(\frac{333}{5218} - 0.07)}{\sqrt{\dfrac{(.07)(.93)}{5218}}} \approx -1.750 < -1.6449$ (in C.R.)

Decision: Reject H_0 in favour of H_1

Conclusion: The study provided enough evidence to indicate that the new system is effective in reducing no-shows.

16. H_0: $\mu \leq 90{,}000$ mi. where μ = mean distance traveled by buses before
H_1: $\mu > 90{,}000$ mi. a major engine failure.

Test statistic: $z = \dfrac{\bar{x} - 90000}{s/\sqrt{n}}$

Critical region: $z > z_{.05} =$ invNorm(.95, 0, 1) ≈ 1.6449

Evaluate test statistic: $z = \dfrac{96700 - 90000}{37500/\sqrt{191}} \approx 2.4692 > 1.6449$ (in C.R.)

Decision: Reject H_0 in favour of H_1

Conclusion: The study supports the manufacturer's claim that the mean distance before a major engine failure exceeds 90000 mi.

17. a) $p < 0.2$ b) $p \geq 0.2$
c) $H_0: p \geq 0.2$ d) $H_1: p < 0.2$
e) We have concluded that $p < 0.2$; i.e., that less than 20% of BC voters will vote NDP.

18. $n = 16$; $\bar{x} = 3.6750$; $s \approx 0.657318$

H_0: $\mu = 3.39$ kg where μ = mean birth weight of males whose mother
H_1: $\mu \neq 3.39$ kg used a special vitamin supplement.

Test statistic: $t = \dfrac{\bar{x} - 3.39}{s / \sqrt{n}}$

Critical region: $t < -t_{.025}$ or $t > t_{.025}$ (15 degrees of freedom);
i.e., $t < -2.132$ or $t > 2.132$

Evaluate test statistic: $t = \dfrac{3.675 - 3.39}{.657318 / \sqrt{16}} \approx 1.7343$ (Not in C.R.)

Decision: Fail to reject H_0

Conclusion: At the 0.05 level there is insufficient evidence to conclude that the vitamin supplement has an effect on the birth weight of male babies.

19. a) $\mu < 10$ pounds b) H_0: $\mu \geq 10$ pounds
c) H_1: $\mu < 10$ pounds d) left-tailed
e) Concluding that the mean weight of paper discarded per week by North Van households is less than 10 pounds when in fact it is not.
f) The sample evidence was not strong enough to conclude that the mean weight of paper discarded per week by North Van households is less than 10 pounds.

20. H_0: $\mu = 41$ points where μ = mean anxiety score of Egyptian high school students
H_1: $\mu > 41$ points

Test statistic: $z = \dfrac{\bar{x} - 41}{s / \sqrt{n}}$

Critical region: $z > z_{.01}$ = invNorm(.99, 0, 1) ≈ 2.3263

Evaluate test statistic: $z = \dfrac{50 - 41}{12.7 / \sqrt{277}} \approx 11.7945 > 2.3263$ (in C.R.)

Decision: Reject H_0 in favour of H_1

Conclusion: The sample data supports the researchers' theory that Egyptian high school students experience higher anxiety levels on average.

21. H_0: $p = 0.15$ where p is the proportion of drivers in the country who have tampered
H_1: $p \neq 0.15$ with their emission control devices.

Test statistic: $z = \dfrac{\hat{p} - 0.15}{\sqrt{\dfrac{(0.15)(0.85)}{n}}}$

Critical region: $z < -z_{.025}$ or $z > z_{.025}$; i.e., $z < -1.96$ or $z > 1.96$

Evaluate test statistic: $z = \dfrac{(\frac{21}{200} - 0.15)}{\sqrt{\dfrac{(0.15)(0.85)}{200}}} \approx -1.7823$ (Not in C.R.)

Decision: Fail to reject H_0

Conclusion: The data did not suggest that the proportion of cars with tampered emission control devices differs significantly in this county.

22. a) H_0: $\mu = 8000$ where μ is the mean uptake of radio labeled amino acids for
H_1: $\mu < 8000$ for cultures grown in a nitrate medium.

Test statistic: $t = \dfrac{\bar{x} - 8000}{s / \sqrt{n}}$

Critical region: $t < -t_\alpha$ (14 degrees of freedom), or $t < -1.761$, assuming $\alpha = 0.05$

Evaluate test statistic: $t = \dfrac{7788.8 - 8000}{1002.4308 / \sqrt{15}} \approx -0.8160$ (Not in C.R.)

Decision: Fail to reject H_0

Conclusion: The data evidence is not strong enough to conclude that the addition of nitrates results in a decrease of amino acid uptake.

b) Assume that the population of amino acid uptakes is normally distributed (and that a significance level of 0.05 is to be used), and that the 15 results represent a random sample of all amino acid uptakes.

23. H_0: $\mu = 80$ km/h where μ is the average speed of cars on the Squamish highway.
H_1: $\mu > 80$ km/h

Test statistic: $t = \dfrac{\bar{x} - 80}{s / \sqrt{n}}$

Critical region: $t > t_{.05}$ (19 degrees of freedom), or $t > 1.729$

Evaluate test statistic: $t = \dfrac{85 - 80}{10 / \sqrt{20}} \approx 2.2361 > 1.729$ (in C.R.)

Decision: Reject H_0 at the 0.05 level

Conclusion: The data provided sufficient evidence to conclude that, on the average, cars speed (i.e., exceed 80 km/h) on the Squamish highway.

24. H_0: $p = 0.25$ where p is the proportion of sexually active teens that practice "safe sex"
H_1: $p \neq 0.25$

Test statistic: $z = \dfrac{\hat{p} - 0.25}{\sqrt{\dfrac{(0.25)(0.75)}{n}}}$

Critical region: $z < -z_{.005}$ or $z > z_{.005}$; i.e., $z < -2.5758$ or $z > 2.5758$

Evaluate test statistic: $z = \dfrac{(\frac{280}{1050} - 0.25)}{\sqrt{\dfrac{(0.25)(0.75)}{1050}}} \approx 1.2472$ (Not in C.R.)

Decision: Fail to reject H_0

Conclusion: The data does not provide strong evidence against the researcher's claim; i.e., there is not strong evidence to suggest that the percentage of sexually active teens that practice safe sex is not 25%.

25. H_0: $\mu = 3.1$hours where μ is the mean lifetime of Duracell's AAA batteries.
H_1: $\mu > 3.1$hours

Test statistic: $t = \dfrac{\bar{x} - 3.1}{s / \sqrt{n}}$

Critical region: $t > t_{.05}$ (5 degrees of freedom), or $t > 2.015$

Evaluate test statistic: $t = \dfrac{3.35 - 3.1}{0.187083 / \sqrt{6}} \approx 3.2733 > 2.015$ (in C.R.)

Decision: Reject H_0 at the 0.05 level

Conclusion: The data supports Duracell's claim that the mean lifetime of their AAA batteries exceeds 3.1 hours.

26. H_0: $p \geq 0.35$ where p is the proportion of theatre goers that buy food or drink.
H_1: $p < 0.35$

Test statistic: $z = \dfrac{\hat{p} - 0.35}{\sqrt{\dfrac{(0.35)(0.65)}{n}}}$

Critical region: $z < -z_{.05} \approx -1.6449$

Evaluate test statistic: $z = \dfrac{(\frac{38}{120} - 0.35)}{\sqrt{\dfrac{(.35)(.65)}{120}}} \approx -0.7656$ (Not in C.R.)

Decision: Fail to reject H_0 at the 0.05 level

Conclusion: The sample data did not provide strong evidence that the percentage of theatre goers that buy food or drink is less than 35%.

27. H_0: $\mu = \$1352.45$ where μ = average savings account amount.
H_1: $\mu \neq \$1352.45$

Test statistic: $t = \dfrac{\bar{x} - 1352.45}{s/\sqrt{n}}$

Critical region: $t < -t_{.005}$ or $t > t_{.005}$ (24 degrees of freedom); i.e., $t < -2.797$ or $t > 2.797$

Evaluate test statistic: $t = \dfrac{1478.34 - 1532.45}{246.28/\sqrt{25}} \approx 2.5558$ (Not in C.R.)

Decision: Fail to reject H_0 at the 0.01 level

Conclusion: The sample data did not provide strong evidence that the average savings account amount differs from \$1352.45.

28. a) H_0: $\mu \leq 7$ hours/week where μ is the mean time spent partying
H_1: $\mu > 7$ hours/week for students at the university.

Test statistic: $z = \dfrac{\bar{x} - 7}{s/\sqrt{n}}$

Critical region: $z > z_{.05} = \text{invNorm}(.95, 0, 1) \approx 1.6449$

Evaluate test statistic: $z = \dfrac{10.7 - 7}{2.3/\sqrt{35}} \approx 9.5172$ (in C.R.)

Decision: Reject H_0 in favour of H_1 at the 0.05 level

Conclusion: The sample evidence is strong enough to conclude that the average time spent partying is more than 7 hours per week.

b) i) On the basis of the evidence in the sample, we would conclude that the average time spent partying is greater than 7 hrs/wk when in fact it is not.

ii) We would conclude that there is not enough evidence in the sample to indicate that the average time spent partying is greater than 7 hrs/wk, when in fact the average is.

29. H_0: $p = 0.40$ where p is the present proportion of adults who regularly exercise.
H_1: $p \neq 0.40$

Test statistic: $z = \dfrac{\hat{p} - 0.40}{\sqrt{\dfrac{(0.40)(0.60)}{n}}}$

Critical region: $z < -z_{.025}$ or $z > z_{.025}$; i.e., $z < -1.96$ or $z > 1.96$

Evaluate test statistic: $z = \dfrac{(\dfrac{470}{1000} - 0.40)}{\sqrt{\dfrac{(0.40)(0.60)}{1000}}} \approx 4.5185$ (in C.R.)

Decision: Reject H_0 in favour of H_1

Conclusion: The sample evidence is strong enough to conclude that the proportion of adults that exercise regularly has changed since 1985.

30. H_0: $\mu \geq 12.5$ months where μ is the mean time when children start walking.
H_1: $\mu < 12.5$ months

Test statistic: $t = \dfrac{\overline{x} - 12.5}{s / \sqrt{n}}$

Critical region: $t < -t_{.01}$ (17 degrees of freedom), or $t < -2.567$

Evaluate test statistic: $t = \dfrac{12.2778 - 12.5}{2.0236 / \sqrt{18}} \approx -0.4659$ (Not in C.R.)

Decision: Fail to reject H_0 at the 0.01 level

Conclusion: The sample evidence is not strong enough to question the psychologist's claim.