Sections ~~4.1~~ – ~~4.2~~ Problems
5.1 5.2

1. a) P(winnings = \$0) = 1 – 1/1000 – 1/200 = 497/500 = 0.994

   b) $E(X) = \sum[x \cdot p(x)] = 290\left(\frac{1}{1000}\right) + 40\left(\frac{1}{200}\right) + 0\left(\frac{497}{500}\right) = 0.49$

   The expected winnings are \$0.49 or 49 cents.

   c) $\sigma = \sqrt{\sum\left[(x-\mu)^2 \cdot p(x)\right]} = \sqrt{(290-.49)^2(.001) + (40-.49)^2(.005) + (0-.49)^2(.994)}$

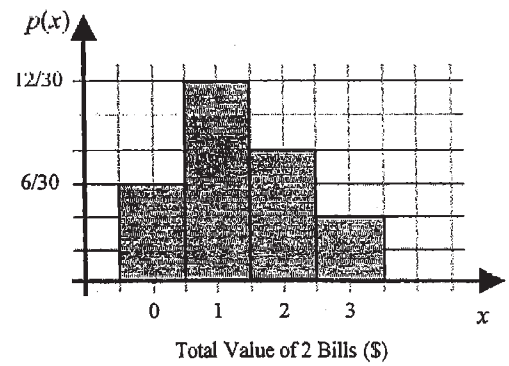
   ≈ 9.584357 or approximately \$9.58.

   d) P(at least one win) = 1 – P(all lose) = $1-(.994)^7 \approx 0.0208$

2. Table:

x	0	1	2	3	Check
$p(x)$	6/30	12/30	8/30	4/30	$\sum p(x) = 1$

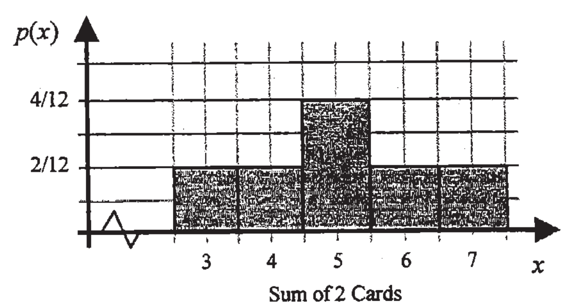
Graph:



3. a) $S = \{(1, 2), (1, 3), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (3, 4), (4, 1), (4, 2), (4, 3)\}$

   b)

x	3	4	5	6	7	Check
$p(x)$	2/12	2/12	4/12	2/12	2/12	$\sum p(x) = 1$



   c) $\mu = \sum[x \cdot p(x)] = 5$ $\sigma = \sqrt{\sum\left[(x-\mu)^2 \cdot p(x)\right]} = 1.290994449 \approx 1.2910$

4.

x	0	1	2	Check
$p(x)$	56/132	64/132	12/132	$\sum p(x) = 1$

5. a) $E(X) = \mu = \sum[x \cdot p(x)] = 0.49$

   b) $\sigma = \sqrt{\sum \left[(x-\mu)^2 \cdot p(x)\right]} \approx 0.9326842981 \approx 0.9327$ (from TI-83)

6. a) $P(\text{winnings} = 0) = 1 - 1/73 - 1/8 = 503/584$

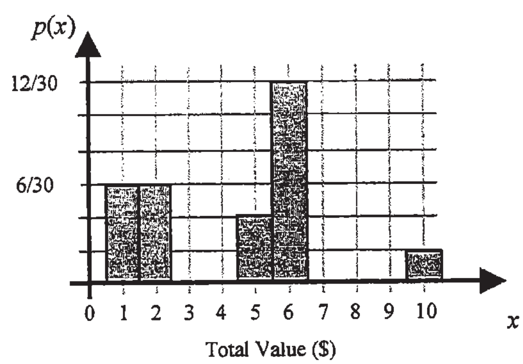
   b) Expected winnings = $\mu = \sum[x \cdot p(x)] = 20(1/73) + 2(1/8) + 0(503/584)$
   $= 153/292 \approx 0.52397 \approx \$0.52$ or 52 cents

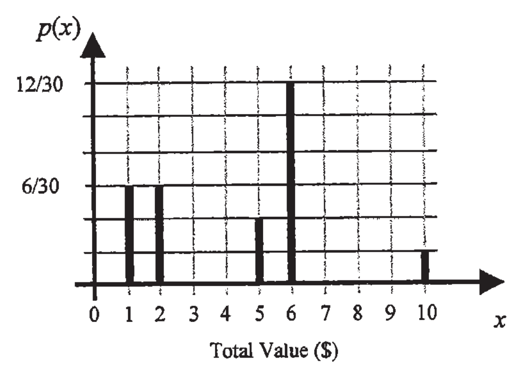
7. a) Table

x	1	2	5	6	10	Check
$p(x)$	6/30	6/30	4/30	12/30	2/30	$\sum p(x) = 1$

Graph

Histogram or Line Graph





   b) With replacement, a total of $0 is now possible.
   Therefore the possible values of X are: $\{0, 1, 2, 5, 6, 10\}$

8. a) $p(3) = 1 - 0.15 - 0.35 = 0.5$
   b) (i) $\mu = \sum[x \cdot p(x)] = 2.2$
   (ii) $\sigma \approx 1.029563014 \approx 1.0296$

9. a) $S = \{(2, 4), (2, 6), (2, 8), (4, 2), (4, 6), (4, 8), (6, 2), (6, 4), (6, 8), (8, 2), (8, 4), (8, 6)\}$

b)

x	3	4	5	6	7	Check
$p(x)$	2/12	2/12	4/12	2/12	2/12	$\sum p(x) = 1$

10. a)

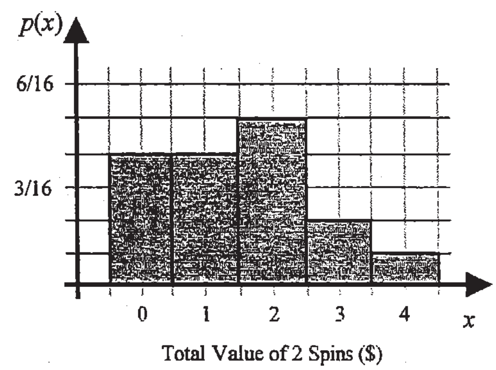
x	100	10	2	−5	Check
$p(x)$	1/52	3/52	12/52	36/52	$\sum p(x) = 1$

b) Expected winnings = $E(X) = \mu = \sum[x \cdot p(x)] = -26/52 = -0.5$
This is not a good game to play, because "on the average" you expect to lose \$0.50 or 50 cents every time you play.

11. Table

x	0	1	2	3	4	Check
$p(x)$	4/16	4/16	5/16	2/16	1/16	$\sum p(x) = 1$

Graph



12. a) $\mu = \sum[x \cdot p(x)] = 1.6$ defects
b) $\sigma = \sqrt{\sum\left[(x-\mu)^2 \cdot p(x)\right]} \approx 1.067707825 \approx 1.0677$ defects (from TI-83)

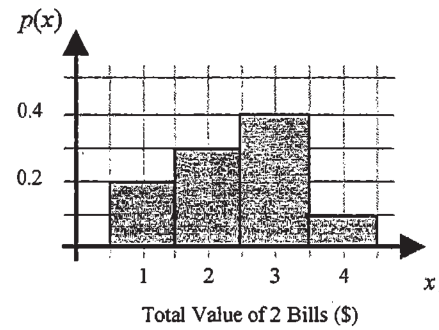
13. a) $P(X \geq 2) = 0.15 + 0.05 + 0.05 = 0.25$
b) $\mu = \sum[x \cdot p(x)] = 1.05$ claims
c) $\sigma = \sqrt{\sum\left[(x-\mu)^2 \cdot p(x)\right]} \approx 1.071214264 \approx 1.0712$ claims (from TI-83)

14. $P(\text{winnings} = 5) = 1 - 4/7 - 1/5 = 8/35$

15. a) Table

x	1	2	3	4	Check
$p(x)$	4/20	6/20	8/20	2/20	$\sum p(x) = 1$

Graph



b) No, a total value of 0 would now be possible as well.

16. $E(X) = \sum[x \cdot p(x)] = 500000(1/3764376) + 1000(1/9906) + 10(1/141) + 1(1/6.8) + 0(p(0))$

$\approx 0.4517538679 \approx \$0.45$ or 45 cents

The expected winnings for each \$1 played are \$0.45.

17. a) $P(X \geq 2) = 0.2 + 0.15 + 0.05 = 0.40$

b) $\mu = \sum[x \cdot p(x)] = 1.3$ passengers

b) $\sigma = \sqrt{\sum \left[(x-\mu)^2 \cdot p(x)\right]} \approx 1.228820573 \approx 1.2288$ passengers (from TI-83)

18. $p(0) = 0$; $p(1) = 1/4$; $p(2) = 2/4$; $p(3) = 3/4$

All values of $p(x) > 0$, but $\sum p(x) = 6/4 = 1.5 \neq 1$

No, this is not a probability distribution because all the probabilities add up to more than 1.

19. Expected winnings = $175(1/38) - 5(37/38) = -10/38 \approx -0.26$

Therefore, the expected net winnings are \$– 0.26; i.e. expect to lose \$0.26 "on the average".

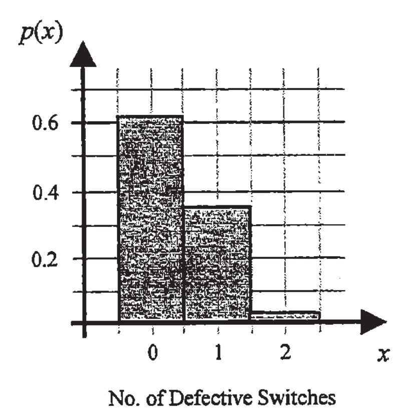
20. a) $p(4) = 1 - 42/125 - 118/250 - 1/10 - 1/25 = 13/250$

b) $E(X) = \sum[x \cdot p(x)] = 1$ woman

21. a) Table

x	0	1	2	Check
$p(x)$	56/90	32/90	2/90	$\sum p(x) = 1$

Graph



b) Mean = $\mu = \sum[x \cdot p(x)] = 0.4$ switches

Variance $\sigma^2 = \sum \left[(x-\mu)^2 \cdot p(x)\right] = 0.28\dot{4}$ (switches)^2

SD = $\sigma = \sqrt{0.28\dot{4}} = 0.5\dot{3}$ switches