Chapter 3 answers

Section ~~2.4 – 2.7~~ Problems 3.2, 3.3, 3.4

1. a) (i) From calculator: $\bar{x} = 270.125$; mean sale price = \$270,125

   (ii) From calculator: $s \approx 155.8400622$; sale price SD ≈ \$155,840

   b) Position of Q_3: $0.75(8) = 6$; take average of $6^{th} + 7^{th}$ item
   Value of $Q_3 = (320 + 350)/2 = 335$ or \$335,000
   (same answer whether using above "classroom method" or TI-83 value for Q_3)

   c) Position of $P_{30} = 0.30(8) = 2.4$, or 3^{rd} item
   Value of $P_{30} = 157$ or \$157,000

2. a) Mean $\bar{x} = \dfrac{\sum(x \cdot f)}{n} = \dfrac{31}{40} = 0.775$; i.e., mean ≈ 0.8 cups

   b) Median $\tilde{x}$ = average of 20^{th} & 21^{st} items = 0 cups

   c) $s \approx 1.349$ (from calculator); i.e., SD ≈ 1.3 cups

3. a) Mean $\bar{x} = \dfrac{\sum(x \cdot f)}{n}$; using the class marks (2.5, 5.0, 7.5, 10) for x,
   $\bar{x} = 6.9375$ (from calculator); i.e., $\bar{x} \approx 6.9$ hours

   b) $s \approx 1.649349133$ (from calculator)
   variance $= s^2 \approx 1.649349^2 \approx 2.72035$; i.e., $s^2 \approx 2.7$ hours^2

4. a) \$400,000

   b) Q_1 of final prices ≈ \$150,000

   c) SD $\approx \dfrac{Range}{4} = \dfrac{\max - \min}{4} = \dfrac{1300 - 200}{4} = 275$; i.e., $s \approx \$275,000$

5. Standard score $= z = \dfrac{x - \bar{x}}{s} = \dfrac{14 - 16.5}{2} = -1.25$

6. $n = 10$; median is average of 5^{th} and 6^{th} item; $\tilde{x} = \dfrac{1800 + 2200}{2} = \$2000$

7\. a) $\bar{x} = \dfrac{\sum x}{n} = \dfrac{24}{6} = 4$ goals

b) (i) $\Sigma x^2 = 11^2 + 4^2 + 6^2 + 0^2 + 0^2 + 3^2 = 182$

(ii) $(\Sigma x)^2 = 24^2 = 576$
(iii) $\Sigma(x - \bar{x}) = 0$ (this is always true for any data set)

c) $\text{Variance} = \dfrac{\sum (x-\bar{x})^2}{n-1} = \dfrac{7^2 + 0^2 + 2^2 + (-4)^2 + (-4)^2 + (-1)^2}{5} = 17.2$

$\text{SD} = \sqrt{\text{variance}} = \sqrt{17.2} \approx 4.1$ goals;

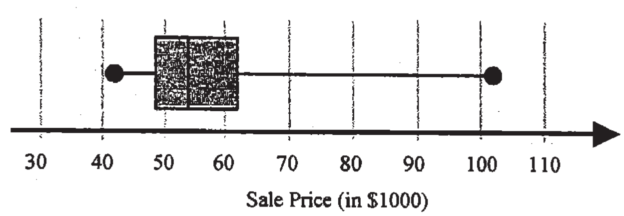
or, from calculator, $s \approx 4.1$ goals

8\. a) modc = 0 passengers
b) median $\tilde{x} = 1$ passenger
c) From calculator, $\bar{x} = 1.18$; i.e., mean ≈ 1.2 passengers
d) From calculator, s ≈ 1.172647 ; i.e., SD ≈ 1.2 passengers

9\. $\text{SD} \approx \dfrac{Range}{4} = \dfrac{\text{max} - \text{min}}{4} \approx \dfrac{10-2}{4} = 2$ hours
(Answers may vary; I'm guessing that the "longest sleep" and "shortest sleep" were approximately 10 and 2 hours, respectively.)

10\. a) Minimum = \$42,500 first quartile Q_1 = \$47,600 median $\tilde{x}$ = \$53,500 ; third quartile Q_3 = \$62,000 maximum = \$102,800

b)



c) $Q_3 - Q_1$ = \$14,400 d) Q_3 = \$62,000

Note : The above quartile calculations were done using the "classroom method". If the TI-83 is used for the quartile calculations, the results differ slightly: Q_1 = \$47,300 ; Q_3 = \$62,900 ; the boxplot will not be noticeably affected.)

11. a) $z = \frac{x - \bar{x}}{s} = \frac{65-70}{10} = -0.5$ b) $z = \frac{455-500}{80} = -0.5625$

Conclusion: The first score (-0.5) is a better score because it is higher relative to the other scores. (This assumes that "high" scores are "good" scores.)

12. a) Position of $P_{70} = 0.70(10) = 7$; i.e., average of 7th + 8th item

Value of $P_{70} = \frac{10.78 + 10.79}{2} = 10.785$ sec.

b) Position of $Q_1 = .25(10) = 2.5$; take 3rd item.

Value of $Q_1 = 10.73$ sec.

13. a) $\tilde{x} = 10.6$ sec. Actually 11.6 s

b) (Note: "Faster" means "lower" times!)

$Q_3 \approx 10.5$ sec. Actually Q3 = 11.5 s

14. a) $\bar{x} = 1.9\%$ b) $\tilde{x} = 1.9\%$ c) $s \approx 0.47\%$

15. mean $\bar{x} = 1.45$ or ≈ 1.5 cups

median $\tilde{x} = 1$ cup

mode = 0 cups

SD $s \approx 1.38300$ or ≈ 1.4 cups

16. Using the class marks for each class as x (18, 21, 24, 27):

a) mean $\bar{x} = 21.75$ or ≈ 21.8 years

b) variance $s^2 \approx (2.941742)^2 \approx 8.7$ years^2

c) SD $s \approx 2.94$ or ≈ 2.9 years

17. σ

18. It is the "balancing point" of the histogram that describes the distribution of values.

19. The deviations $(x - \bar{x})$ of the 3 values were 8, –4, –4.

$$\text{SD} = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}} = \sqrt{\frac{8^2 + (-4)^2 + (-4)^2}{2}} = \sqrt{\frac{64+16+16}{2}} \approx 6.9 \text{ min.}$$

20. a) $\frac{3}{4}=1-\frac{1}{4}=1-\frac{1}{2^2}=1-\frac{1}{k^2}$; here, $k=2$

at least ¾ of the date lies within 2 SD's of the mean; i.e., between $\bar{x}-2s$ and $\bar{x}+2s$, or $62-2(6)$ and $62+2(6)$ or between 50 and 74.

b) minimum $\approx \bar{x}-2s=50$ maximum $\approx \bar{x}+2s=74$

c) $z=\frac{x-\bar{x}}{s}=\frac{80-62}{6}=3$

This is considered to be "unusual" since it is > 2.

21. a) $\bar{x}=9$; $\tilde{x}=9$; the modes are 7 and 10 (bimodal!)

b) position of $Q_1 = .25(9) = 2.25$; i.e., take 3rd value

Value of $Q_1 = 7$

c) $s \approx 3.4641016$ or $s \approx 3.5$ (from calculator)

22. a) Mean $\bar{x} = 5$

Mean deviation = $\frac{\sum|x-\bar{x}|}{n}=\frac{|6-5|+|3-5|+|7-5|+|7-5|+|4-5|+|5-5|}{5}$

$=\frac{1+2+2+1+0}{5}=\frac{6}{5}=1.2$

b) Variance $=\frac{\sum(x-\bar{x})^2}{n-1}=\frac{1^2+(-2)^2+2^2+(-1)^2+(0)^2}{4}=2.5$

c) $s=\sqrt{s^2}=\sqrt{2.5}\approx 1.58114 \approx 1.6$

23. a) Class width = 5 years b) $\bar{x}\approx 22$ years (Using Calculator: 1-Var Stats L_1,L_2)

c) For a grouped frequency table, we do not know the actual data values. We have replaced all the values in a class by the appropriate class mark, which may not equal the actual data values. Hence, the calculated mean may not be the actual mean.

d) $s=\sqrt{\frac{\sum\left[(x-\bar{x})^2\cdot f\right]}{n-1}}=\sqrt{\frac{(12-22)^2(2)+(17-22)^2+(22-22)^2(3)+(27-22)^2(5)}{10}}$

$=\sqrt{35}\approx 5.9$ years

24. a) (i) $\bar{x} = \$393,583$ (ii) $s \approx \$25,969$

b) $Q_1 = \$376,250$

c) $P_{70} = \$410,000$

d) $s \approx \frac{Range}{4} = \frac{\max - \min}{4} = \frac{2,388,888 - 357,000}{4} = \$507,972$

25. a) mode = 5 BR b) $\tilde{x} = \frac{4+5}{2} = 4.5$ BR

c) $\bar{x} = 4.30556$ or $\approx$ 4.3 BR

26. a) $\bar{x} = 65.7$ strokes b) $\tilde{x} = 66$ strokes

c) $s \approx 1.1188 \approx 1.1$ strokes

d) Sum of the squares $= \sum(x^2 f) = 63^2(1) + 64^2(4) + 65^2(6) + 66^2(11) + 67^2(8) = 129,531$

e) Since $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$, the sum of the squares of the deviations is

$\sum(x-\bar{x})^2 = s^2(n-1) \approx 1.1188^2(29) \approx 36.3$

27. Eric's standard score $z = \frac{x-\bar{x}}{s} = \frac{88-72}{12} = 1.3$

Andrea did better, because her standard score (1.5) was higher.

28. (a) $\bar{x} \approx 12.6$ goals (b) $\tilde{x} = 11.5$ goals (c) $s \approx 5.9$ goals

29. Probably the "PIM" data, since it seems to have the largest range (max-min) of the data sets, and $s \approx$ Range/4.

30. Total scores = 7 (15) + 11 = 116

Average score $= \bar{x} = \frac{\text{Total}}{8} = \frac{116}{8} = 14.5$ points

31. a) $\bar{x} = \frac{\sum x}{n}$ or $\frac{\sum (xf)}{n} = \frac{0(5)+1(13)+2(12)+3(10)}{40} = \frac{67}{40} = 1.675 \approx 1.7\text{cards}$

b) $\tilde{x} = 2$ cards   c) mode = 1 card

d) SD = s

$$= \sqrt{\frac{\sum (x-x)^2 f}{n-1}} \approx \sqrt{\frac{(0-1.675)^2(5)+(1-1.675)^2(13)+(2-1.675)^2(12)+(3-1.675)^2}{39}}$$

$\approx 0.997111 \approx 1.0$ cards

32. $SD \approx \frac{\text{Range}}{4} = \frac{\text{max} - \text{min}}{4} = \frac{20-4}{4} = 4$

(Answers may vary; I guessed that the max and min quiz scores would be 20 and 4, respectively.)

33. a) $\bar{x} = 11$ hours   b) $s \approx 5.6$ hours

34. a) (i) $\bar{x} = \$557.50$ (ii) $\tilde{x} = \$545$ (iii) M = \$500
(iv) MR = \$550

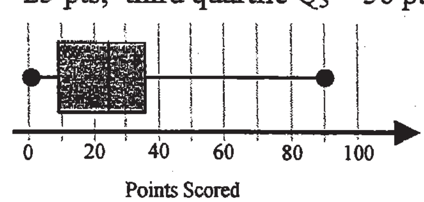
b) No change in median and mode; new mean is \$574.20 (up by \$16.70)
new MR = \$625 (up by \$75)
The MR changes most.

35. a) Location of $P_{60} = 0.60(15) = 9$; take average of 9th + 10th item
Value of $P_{60} = \frac{104+105}{2} = \$104.50$

b) Location of $P_{35} = 0.35\ (15) = 5.25$; take 6th item
Value of $P_{35} = \$89$

36. $z = \frac{x-\bar{x}}{5} = \frac{1.2-2.5}{0.5} = -2.6$

37. a) min = 1 pt; first quartile $Q_1 = 9$ pts; median $\tilde{x} = 25$ pts; third quartile $Q_3 = 36$ pts; max = 90 pts
b) skewed right
c) Expect the mean to be greater than the median



38. a) Total = 0(12) + 1(2) + 2(3) + 3(2) + 4(1) = 18

b) Mean = $\bar{x} = \frac{\sum x}{n} = \frac{18}{20} = 0.9$ days

c) Median = 0 days

39. a) Mean = $\bar{x} = 16.5$

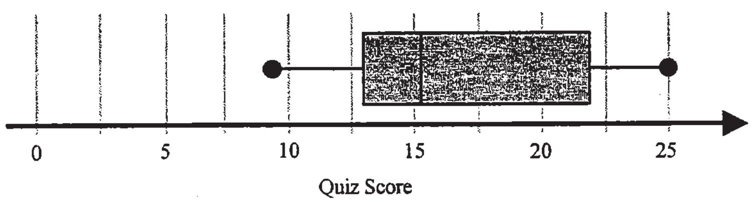
b) Median $\tilde{x} = \frac{14+17}{2} = 15.5$

c) Modes are 13 and 22

d) $s \approx 5.5$

e) $Q_1 = 13$; $Q_3 = 22$

f)



40. a) 332 is $\frac{332-370}{19} = -2$, or 2 SD's below the mean

408 is $\frac{408-370}{19} = +2$, or 2 SD's above the mean.

i.e., between 332 and 408 is within 2 SD's of the mean; Chebyshev says that at least $\left(1-\frac{1}{2^2}\right)100\% = 75\%$ of the data lies in that interval.

b) 408

41. a) (i) Range $\approx$ 40-21 = 19, or $19,000

(ii) Midrange = $\frac{\text{max+ min}}{2} \approx \frac{40+21}{2} = 30.5$, or $30,500

b) (i) mean $\bar{x}$ = 32, or $32,000

(ii) SD $s \approx 3.57192$, or ≈ $3572

c) The range and the standard derivation.

42. a) (i) $P_{80} = \frac{26.1+35.8}{2} = 30.95$, or ≈ $30.95 million

(ii) $Q_3 = 7.6$ or $7.6 million

b) Q_3 = 87.9 or $87.9 million

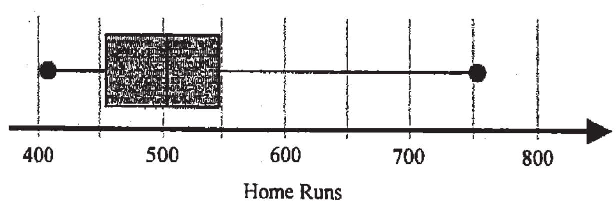
43. Between 2.4 and 4.4 is with 1 kg of the mean; i.e., within 2 SD's of the mean. For a bell-shape histogram, approximately 95% of the data is with 2 SD's of the mean. (68-95-99 Rule, or Empirical Rule)

44. a) $z = \frac{x - \bar{x}}{s} = \frac{227 - 180}{35} \approx 1.34$

b) $z = \frac{418 - 350}{50} \approx 1.36$ Conclusion: The second score has a better z score.

45. a) $P_{40} = \frac{109.99 + 119.99}{2} = 114.99$ b) $P_{65} = 139.99$

46. minimum = 407 ; $Q_1 = 458$; $\tilde{x} = 504$; $Q_3 = 548$; maximum = 755



47. a) The four given numbers are deviations from the mean, $(x - \bar{x})$;
11.50, –3.35, –5.20, 1.10 . These add to 4.05.
Since, for any data set, $\sum (x - \bar{x}) = 0$, the fifth deviation must be –4.05 .
Therefore, Jason's costs were below average by $4.05.

b) $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{(11.5)^2 + (-3.35)^2 + (-5.20)^2 + (1.10)^2 + (-4.05)^2}{4}} \approx \$6.86$

48. a) Sum of all deviations must equal 0 ; missing deviation = 8

b) Variance $= \frac{\sum (x - \bar{x})^2}{n-1} = \frac{(-2)^2 + 5^2 + (-7)^2 + (-4)^2 + (8)^2}{4} = 39.5$

49. a) modal response = "not very" b) median response = "somewhat"

50. Total of 11 quizzes = 11 (14) = 154
Total of "best 10" = 154 – 8 = 146
Mean of "best 10" = 146/10 = 14.6

51. a) Mean = $\frac{\sum x}{n} = \frac{\sum (xf)}{n} = \frac{0(11)+1(8)+2(4)+3(2)}{25} = \frac{22}{25} = 0.88$ pass. or ≈ 0.9 pass.

b) Median $\tilde{x} = 1$ pass.

c) Mode M = 0 pass.

d) $\text{SD} = \sqrt{\frac{\sum (x-x)^2 f}{n-1}} = \sqrt{\frac{(0-.88)^2(11)+(1-.88)^2(8)+(2-.88)^2(4)+(3-.88)^2(2)}{24}}$

≈ 0.97 passengers or ≈ 1.0 pass.

52. a) $\bar{x} \approx 11.5$ sec. b) $s \approx 0.5166759 \approx 0.5$ sec.

53. a) $\bar{x} = 20.8$ years

b) $\tilde{x} = 21$ years

c) Modes are 18 and 22 yrs.

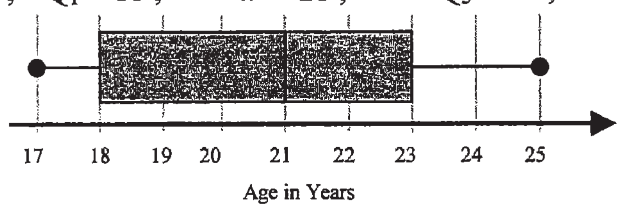
d) $s \approx 2.780887 \approx 2.8$ years

e) Variance = $s^2 \approx 2.780887^2 \approx 7.7$ years^2

f) $Q_1 = 18$; $Q_3 = 23$; Interquartile range = $Q_3 - Q_1 = 5$ years

g) Range = 8 years

h) min = 17 ; $Q_1 = 18$; $\tilde{x} = 21$; $Q_3 = 23$; max = 25



54. a) $\frac{8}{9} = 1 - \frac{1}{9} = 1 - \frac{1}{3^2}$; at least $\frac{8}{9}$ of the salaries are within 3 SD's of the mean;
i.e., between $\bar{x} - 3s$ and $\bar{x} + 3s$; i.e., between \$47,700 and \$67,500.

b) \$48,000 is more than 2 SD's below the mean ($\bar{x} - 2s = 51{,}000$); it is unusually low.

55. a) 10 b) 60

c) The ranges of values are approximately equal (≈ 105) for both teams; however, for the Maple Leafs, 50% of the data (between Q_1 and Q_3) lies very close to the median; for the Canucks, the middle 50% of the data is much more spread out; I could expect the SD for the Canucks team to be higher.

56. a) (i) $\bar{x} = 2.55 \approx 2.6$ courses (ii) $\tilde{x} = 2.5$ courses (iii) Mode = 3 courses

b) $s = 1.197219 \approx 1.2$ courses.

57. a) $P_{80} = \frac{240+240}{2} = \$240$ b) $P_{30} = \$139$

58. a) $x = \bar{x} + 2.5 = 2.5 - 1.4(0.5) = 1.8$ kg.

b) $z = 1.4$ means 1.4 SD's below the mean. This is not considered to be unusually low.