Sections 4.3 – 4.4 Problems

1. a) (i) The experiment consists of a fixed number of trials.
   (ii) The outcomes of each trial can be classified into two categories (labelled "success" and "failure").
   (iii) The trials are identical and independent.
   (iv) The probabilities of success and failure are constant from trial to trial.
   b) The binomial random variable is the number of successes in n trials.

2. a) 5 040 b) 120 c) $\frac{(100)(99)(98)\cdots(2)(1)}{(98)(97)\cdots(2)(1)\cdot(2)(1)} = \frac{(100)(99)}{(2)(1)} = 4950$

3. $n = 8$; $p = 1/2$; X = number of heads
   a) $P(X = 4)$ = binompdf(8, 0.5, 4) $\approx 0.2734375 \approx 0.2734$
   b) $P(X > 6) = P(X = 6, 7, 8) = 1 - P(X \le 5) = 1 -$ binomcdf(8, 0.5, 5) $\approx 0.14453125 \approx 0.1445$

4. a) No; more than two outcomes per trial
   b) Yes
   c) No; more than two outcomes per trial

5. a) $n = 10$; $p = 0.42$;
   X = number of people in the sample who think that the penalties should be beefed up
   $P(X = 4)$ = binompdf(10, 0.42, 4) ≈ 0.2488
   b) $n = 10$; $p = 0.17$
   X = number of people in the sample who think that the penalty is too severe
   $P(X = 0)$ = binompdf(10, 0.17, 0) ≈ 0.1552

6. $n = 200$; $p = 0.60$;
   X = number of people in the sample who experience pain relief; then X is a binomial r.v.
   a) (i) $\mu = np = 200(0.6) = 120$ people
   (ii) SD = $\sigma = \sqrt{npq} = \sqrt{200(0.6)(0.4)} \approx 6.9282$ people
   b) 110 is 10 below the mean of 120; this is less than 2SD's (≈ 14) below the mean, so it is not considered to be unusually low. There is not strong evidence to doubt the 60% claim.

7. $n = 6$; $p = 0.60$;
   X = number of shots that he sinks in 6 attempts
   a) $P(X = 0)$ = binompdf(6, 0.60, 0) ≈ 0.0041
   b) $P(X \le 2)$ = binomcdf(6, 0.6, 2) ≈ 0.1792
   c) P(missing 3, 4, 5, or 6) = P(sinking 0, 1, or 2) = $P(X \le 2)$ = binomcdf(6, 0.6, 2) ≈ 0.1792

   Assumption: each free shot that he takes is independent of previous free shots; the probability of sinking a free shot is the same (0.6) for each attempt.

8. a) $n = 10$; $p = 0.1$; X = number of correct guesses in 10 trials
   $P(X = 1)$ = binompdf(10, 0.1, 1) ≈ 0.3874
   b) $n = 15$; $p = 0.1$ X = number of correct guesses in 15 trials
   $P(X > 3) = 1 - P(X \leq 3) = 1 -$ binomcdf(15, 0.1, 3) ≈ 0.0556
   c) $n = 1000$; $p = 0.1$ X = number of correct guesses in 1000 trials
   $P(X \leq 120)$ = binomcdf(1000, 0.1, 120) ≈ 0.9827

9. No! If each day is considered a trial, the trials are not independent! The weather on a particular day (rain or not) likely significantly influences the probability of rain on the next day.

10. a) $n = 15$; $p = 0.40$; $q = 0.60$
    X = number of people in the sample with deficient reading skills
    (i) $P(X = 4)$ = binompdf(15, 0.40, 4) ≈ 0.1268
    (ii) $P(X \geq 4) = P(X = 4, 5, \ldots, 15) = 1 - P(X \leq 3) = 1 -$ binomcdf(15, 0.4, 3) ≈ 0.9095
    (iii) $P(X \leq 10)$ = binomcdf(15, 0.4, 10) ≈ 0.9907
    b) (i) $\mu = np = 50(0.4) = 20$ people
    (ii) SD = $\sigma = \sqrt{npq} = \sqrt{50(0.4)(0.6)} \approx 3.464 \approx 3.5$ people

11. $n = 10$; $p = 1/3$; $P(X = 4)$ = binompdf(10, 1/3, 4) ≈ 0.2276

12. a) $\mu = np = 900(0.17) = 153$ people
    SD = $\sigma = \sqrt{npq} = \sqrt{900(0.17)(0.83)} \approx 11.3$ people
    b) 140 is 13 below the mean of 153; this is less than 2SD's (≈ 23) below the mean, so it is not considered to be unusually low. Therefore, this is not considered an unusual result.

13. $x = 35$; $n = 50$; $p = 0.6$; $q = 0.4$

14. a) N b) B c) N d) N e) N

15. a) $n = 14$; $p = 1/10 = 0.1$
    (i) $P(X = 2)$ = binompdf(14, 0.10, 2) ≈ 0.2570
    (ii) $P(X \leq 1)$ = binomcdf(14, 0.10, 1) ≈ 0.5846
    b) $n = 100$;
    $P(X = 4)$ = binompdf(100, 0.10, 4) ≈ 0.0159

16. $n = 8$; $p = 1/2$; X = number of correct guesses in 8 questions

a) $P(X = 3)$ = binompdf(8, 0.5, 3) ≈ 0.2188

b) $P(X \leq 2)$ = binomcdf(8, 0.5, 2) ≈ 0.1445

17. $n = 20$; $p = 0.02$; X = number of carriers in 20 bites

$P(X \geq 1) = P(X = 1, 2, \ldots, 20) = 1 - P(X = 0) = 1 -$ binompdf(20, 0.02, 0) ≈ 0.3324

18. a) B b) B c) B d) Not

Each trial (roll) has more than two outcomes, and the variable "total of all outcomes" cannot be considered to be the "number of times" that a particular outcome on a trial occurs.

19. $n = 50$; $x = 8$; $p = 0.15$; $q = 0.85$

20. $n = 20$; $p = 0.10$; X = number of left handed people in a sample of 20

$P(X > 2) = P(X = 3, 4, \ldots, 20) = 1 - P(X \leq 2) = 1 -$ binomcdf(20, 0.10, 2) ≈ 0.3231

21. a) $n = 5$; $p = 0.44$; $P(X = 5)$ = binompdf(5, 0.44, 5) = $(0.44)^5 \approx 0.0165$

b) $n = 5$; $p = 0.06$; $P(X = 0)$ = binompdf(5, 0.06, 0) = $(0.94)^5 \approx 0.7339$

22. $n = 48$; $p = 1/4 = 0.25$;

a) (i) $\mu = np = 48(0.25) = 12$

(ii) SD = $\sigma = \sqrt{npq} = \sqrt{48(0.25)(0.75)} = 3$

b) 4 is 8 below the mean of 12; this is more than 2SD's (6) below the mean of 12, and hence it can be thought of as unusually low. I would be suspicious about the fairness of the pointer.

23. $n = 10$; $p = 0.265$; X = number of growers receiving jail sentences in 10

a) $P(X = 2)$ = binompdf(10, 0.265, 2) ≈ 0.2692

b) $P(X \geq 2) = P(X = 2, 3, \ldots, 10) = 1 - P(X \leq 1) = 1 -$ binomcdf(10, 0.265, 1) ≈ 0.7881

24. a) $P(0) = 1 - 1/1000 - 1/200 = 0.994$

b) $n = 7$; p = the probability of winning a prize with a ticket = 0.006

X = number of tickets that win a prize

$P(X \geq 1) = P(X = 1, 2, \ldots, 7) = 1 - P(X = 0) = 1 -$ binompdf(7, 0.006, 0) ≈ 0.0413

c) p = the probability of *not* winning a prize with a ticket = 0.994

$n = 365$; $\mu = np = 365(0.994) = 362.81$ tickets

25. $n = 10$; p = the probability of graduating = 0.4
X = number of students in the sample that will graduate

a) $P(X = 10)$ = binompdf(10, 0.4, 10) ≈ 0.0001049

b) $P(X = 0)$ = binompdf(10, 0.4, 0) ≈ 0.006047

c) $P(X = 1)$ = binompdf(10, 0.4, 1) ≈ 0.04031

d) $P(X \geq 1) = P(X = 1, 2, \ldots, 10) = 1 - P(X = 0) = 1 -$ binompdf(10, 0.4, 0) ≈ 0.9940

26. a) $n = 8$; $p = 0.18$
X = number of patients in the sample that experience a complication

(i) $P(X = 2)$ = binompdf(8, 0.18, 2) ≈ 0.2758
(ii) $P(X \geq 1) = P(X = 1, 2, \ldots, 10) = 1 - P(X = 0) = 1 -$ binompdf(8, 0.18, 0) ≈ 0.7956

b) $n = 50$; mean = $\mu = np = 50(0.18) = 9$ patients
SD = $\sigma = \sqrt{npq} = \sqrt{50(0.18)(0.82)} \approx 2.72$ patients

c) 15 is 6 above the mean of 9; this is more than 2SD's (≈ 5.4) above the mean of 9, and therefore it can be thought of as unusually high. Thus, there is cause for concern about the high number of complications.

27. a) $n = 12$; $p = 0.4$
X = number of answered calls in the sample of 12 calls
$P(X \geq 4) = P(X = 4, 5, \ldots, 12) = 1 - P(X \leq 3) = 1 -$ binomcdf(12, 0.4, 3) ≈ 0.7747

b) (0.6)(0.6)(0.4) = 0.144

c) $\mu = np$; $20 = n(0.4)$ so $n = 20/(0.4) = 50$ calls

28. $n = 6$; $p = 0.5$ = the probability that the child is a boy
X = number of boys in a family of 6 children

a) $P(X = 3)$ = binompdf(6, 0.5, 3) = 0.3125

b) P(the number of girls ≥ 1) = P(the number of boys ≤ 5)
= $P(X \leq 5)$ = binomcdf(6, 0.5, 5) ≈ 0.9844

c) $P(X \leq 2)$ = binomcdf(6, 0.5, 2) ≈ 0.3438