Sections 7.1 – 7.4 Problems

1. a) $z_{.125}$ = invNorm(.875, 0, 1) ≈ 1.1503 b) $z_{.04}$ = invNorm(.96, 0, 1) ≈ 1.7507
   c) $z_{.075}$ = invNorm(.925, 0, 1) ≈ 1.4395 d) $z_{.005}$ = invNorm(.995, 0, 1) ≈ 2.5758

2. $\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \approx \bar{x} \pm z_{.005}\frac{s}{\sqrt{n}} = 70 \pm 2.5758\left(\frac{20}{\sqrt{200}}\right) \approx 70 \pm 3.643$ or approx. 70 ± 3.6 minutes

   We are 99% confident that the average daily commute time for all Vancouver drivers is between 66.4 minutes and 73.6 minutes.

3. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2$; $z_{\alpha/2} = z_{.025}$ = invNorm(.975, 0, 1) ≈ 1.9600

   $E = 5$ minutes

   $\sigma \approx \frac{\text{Range}}{4} = \frac{3\text{ hours}}{4} = \frac{180\text{ min}}{4} = 45$ minutes

   $n = \left[\frac{1.9600(45)}{5}\right]^2 \approx 311.16$ (round up to nearest integer)

   The minimum required sample size is 312.

4. a) $t_{.025}$ for 14 df ≈ 2.145 b) $t_{.01}$ for 5 df ≈ 3.365
   c) $t_{.005}$ for 9 df ≈ 3.250 d) $t_{.05}$ for 22 df ≈ 1.717

5. a) The population that is being sampled has to have a normal distribution, with an unknown SD that is being estimated by the sample SD.

   b) $\bar{x} = 6.8$ hours, $s \approx 1.4213$ hours

   90% CI for μ is $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$; $t_{\alpha/2} = t_{.05}$ for 4 df ≈ 2.132

   90% CI for μ is $6.8 \pm 2.132\frac{1.4213}{\sqrt{5}} \approx 6.8 \pm 1.35$ or 6.8 ± 1.4 hours

6. To reduce the margin of error from 2 kg to 0.5 kg (i.e., to reduce "by a factor of 4") requires increasing the sample size "by a factor of 4^2", i.e., increasing the sample size from 50 to $50(4^2)$, or 800.

7. Calculations and conclusions are only valid if the 225 visitors to the site can be interpreted as a random sample of all the people who had repair bills for that type of car. It is highly unlikely that the website group of people is a random sample. Therefore, I would not recommend that this interval be published.

8. $E = z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$, so for the same population (and hence, the same σ), $\dfrac{z_{\alpha/2}}{\sqrt{n}}$ equals

   a) $\dfrac{z_{.05}}{\sqrt{100}} \approx 0.1645$ b) $\dfrac{z_{.005}}{\sqrt{200}} \approx 0.1821$

   Therefore, a) gives the smaller margin of error.

9. $n = \dfrac{z_{\alpha/2}^2\hat{p}\hat{q}}{E^2}$; here, $z_{\alpha/2} = z_{.05}$ = invNorm(.95, 0, 1) ≈ 1.6449

   $E = .02;\ \hat{p} = .06,\ \hat{q} = .94$

   $n = \dfrac{(1.6449)^2(.06)(.94)}{.02^2} \approx 381.50$ (round up to nearest integer)

   The minimum required sample size is 382.

10. 0.53 ± 0.0778

11. a) 90% CI for μ is 1.16 ± 0.19 mg.
    b) the mean tar content per Carlton cigarette for all human smokers
    c) We are 90% confident that for human smokers, the mean tar level for Carlton cigarettes lies between 0.97 mg and 1.35 mg per cigarette.
    d) The tar content levels for all Carlton cigarettes for human smokers are normally distributed.
    e) $n = \left[\dfrac{z_{\alpha/2}s}{E}\right]^2 \approx \left[\dfrac{z_{.005}(0.2825)}{0.1}\right]^2 \approx 52.96$; so need a sample size of at least 53.

12. a) $E = z_{.10}\dfrac{3.2}{\sqrt{100}} \approx 0.41$ inches
    b) $2E = 2 \cdot z_{.05}\dfrac{3.2}{\sqrt{100}} \approx 1.05$ inches
    c) $\bar{x} \pm z_{.005}\dfrac{3.2}{\sqrt{100}} = 66 \pm 0.82$ inches

13. a) i) 85% CI for μ is 351.4 ± 0.47 grams
ii) 98% CI for μ is 351.4 ± 0.76 grams
b) 99% CI for μ is 250.8 ± 0.40 grams. The margin of error is $E \approx 0.40$ grams
c) need n to be at least 39;
assume that the population SD can be estimated from the pilot study sample SD in Part b) ($s = 2.2$ grams)

14. a) weights of all mature grizzlies in the park are normally distributed
b) i) 98% CI for μ is 363.2 ± 43.0 kg
ii) 95% CI for μ is 363.2 ± 32.9 kg

15. The minimum n required is 97.

16. a) 196
b) at least 311

17. a) ordinal
b) 99% CI for μ is 4.04 ± 0.18 litres

18. a) 96% CI for the true % of N.S. residents with a university degree is $33 \pm 4\%$.
b) $E \approx 0.0225$
c) at least 8460

19. a) at least 2637
b) at least 1688

20. a) i) 90% CI for μ is $6 \pm z_{.05}\dfrac{2.3}{\sqrt{1000}} \approx 6 \pm 0.12$ hours/week

ii) 98% CI for μ is $6 \pm z_{.01}\dfrac{2.3}{\sqrt{1000}} \approx 6 \pm 0.17$ hours/week

b) 99% CI for μ is $9.0 \pm z_{.005}\dfrac{3.7}{\sqrt{280}} \approx 9.0 \pm 0.57$ hours/week

c) If you were to take many samples of 280 and calculate the CI for each one, you would expect the calculated CI to contain the true population mean about 99% of the time.

21. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2$; $z_{\alpha/2} = z_{.025} = \text{invNorm}(.975, 0, 1) \approx 1.9600$

$E = 10$ minutes

$\sigma \approx \frac{\text{Range}}{4} = \frac{6\text{ hours}}{4} = \frac{360\text{ min}}{4} = 90$ minutes

$n = \left[\frac{1.9600(90)}{10}\right]^2 \approx 311.16$ (round up to nearest integer)

The minimum required sample size is 312.

22. Reduce E from 2.8 to 0.7; i.e., reduce E by a factor of 4, so must increase sample size by a factor of 4^2; i.e., from 100 to $100(4^2)$; need a sample size of 1600.

23. a) The population that is being sampled has to have a normal distribution, with an unknown SD that is being estimated by the sample SD; the sample has to be random.

b) 90% CI for μ is $\bar{x} \pm t_{.05}\frac{s}{\sqrt{n}}$; $t_{.05}$ for 5 df ≈ 2.015

90% CI for μ is $6.9 \pm 2.015\frac{0.898888}{\sqrt{6}} \approx 6.9 \pm 0.74$ seconds

24. 90% CI for p is $\frac{75}{500} \pm z_{.05}\sqrt{\frac{\frac{75}{500}\left(1-\frac{75}{500}\right)}{500}} \approx 0.15 \pm 1.6449\sqrt{\frac{(.15)(.85)}{500}} \approx 0.15 \pm 0.026$

25. Approximately 97%

26. a) $n = \frac{(z_{\alpha/2})^2(.5)(.5)}{E^2} = \frac{(z_{.005})^2(.5)(.5)}{(.03)^2} = 1843.03$; i.e., need at least 1844

b) $n = \frac{(z_{\alpha/2})^2\hat{p}\hat{q}}{E^2} = \frac{(z_{.005})^2(.2)(.8)}{(.03)^2} = 1179.54$; i.e., need at least 1180

27. 90% CI for μ is $13 \pm z_{.05}\frac{3}{\sqrt{80}} \approx 13 \pm 0.55$ kg

28. a) width of CI is $32 - 26 = 6$ kg; so margin of error is $E = 3$ kg

b) reducing the margin of error by a factor of 2 requires increasing the sample size by a factor of 2^2; i.e., from 100 to $100(2^2)$; need a sample size of 400

c) No! We are confident that if we repeated this CI calculation many times, approximately 95% of the time our calculated interval will contain the overall population mean.

29. $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2$; $\quad z_{\alpha/2} = z_{.005} =$ invNorm(.995, 0, 1) ≈ 2.5758

$E = 50$ cents $= \$0.5$

$\sigma \approx \frac{\text{Range}}{4} \approx \frac{10}{4} \approx \$2.5$ (different answers are possible)

$$n = \left[\frac{2.5758(2.5)}{0.5}\right]^2 \approx 165.87$$

The minimum required sample size is 166 (different answers are possible).

30. 95% CI for μ is $\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}$; $\quad t_{.025}$ for 8 df ≈ 2.306

95% CI for μ is $1 \pm 2.306\frac{1.9365}{\sqrt{9}} \approx 1 \pm 1.49$ minutes

Assumption: The population of all time differences is normally distributed.

31. 99% CI for p is $\frac{48}{800} \pm z_{.005}\sqrt{\frac{\frac{48}{800}\left(1-\frac{48}{800}\right)}{800}} \approx 0.06 \pm 0.022$

32. $n = \frac{(z_{\alpha/2})^2(.5)(.5)}{E^2} = \frac{(z_{.025})^2(.5)(.5)}{(.04)^2} \approx 600.32$; i.e., need at least 601

33. $n = \frac{(z_{\alpha/2})^2\hat{p}\hat{q}}{E^2} = \frac{(z_{.05})^2(.1)(.9)}{(.03)^2} \approx 270.55$; i.e., need at least 271

34. a) $E = \$1.72$
    b) 22.35 ± 1.72 dollars

35. a) 95% CI for μ is $\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}}$; $t_{.025}$ for 7 df ≈ 2.365

95% CI for μ is $23.75 \pm 2.365\frac{11.0421}{\sqrt{8}} \approx 23.75 \pm 9.23$ hours/month

b) The population of all leisure times is normally distributed and the sample is random.

c) $n = \left[\frac{z_{.035}\sigma}{E}\right]^2 \approx \left[\frac{1.8119(11.0421)}{2}\right]^2 \approx 100.07$; so need a sample size of at least 101.

36. 90% CI for p is $\frac{14}{50} \pm z_{.05}\sqrt{\frac{\frac{14}{50}\left(1-\frac{14}{50}\right)}{50}} \approx 0.28 \pm 0.10$

We are 90% confident that the proportion of all students who own cars is between 0.18 and 0.38.

37. a) $E = \frac{0.53 - 0.45}{2} = 0.04$

b) The sample proportion is in the middle of the CI, so $\hat{p} = \frac{0.45 + 0.53}{2} = 0.49$

Therefore, (0.49)(100) = 49 students in the sample said that they had a job.

38. $E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \approx z_{.03}\frac{s}{\sqrt{250}} \approx 1.8808\frac{0.388}{\sqrt{250}} \approx 0.046$

Therefore the 94% CI for μ is 2.998 ± 0.046 m^3.

We are 94% confident that the mean monthly gas consumption for all BC households is between 2.952 and 3.044 m^3.

39. The sample size is small; the parent population is normal; the parent population SD is unknown and is estimated by the sample SD.

40. a) $n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2$; $z_{\alpha/2} = z_{.02}$ = invNorm(.98, 0, 1) ≈ 2.0537, $E = 15$ minutes

$\sigma \approx \frac{\text{Range}}{4} = \frac{10-7}{4}$ hours = .75 hours = 45 minutes

$n = \left[\frac{2.0537(45)}{15}\right]^2 \approx 37.96$ (round up to nearest integer)

The minimum required sample size is 38.

b) Estimate $\sigma \approx s = 1.35$ hours = 81 minutes

$n = \left[\frac{2.0537(81)}{15}\right]^2 \approx 122.99$ (round up to nearest integer)

The minimum required sample size is 123.

41. 80% CI for the mean study time is 8.1 ± 2.2 hours.

42. Dividing the margin of error by a factor of 5 is achieved by multiplying the sample size by a factor of 5^2. Therefore, the new sample size is $(150)(25) = 3750$

43. a) 99% CI for p is $\frac{1920}{31000} \pm z_{.005}\sqrt{\frac{\frac{1920}{31000}\left(1-\frac{1920}{31000}\right)}{31000}} \approx 0.0619 \pm 0.0035$

We are 99% confident that the proportion of all Canadians that read the National Post is between 0.0584 and 0.0654.

b) 783

44. a) Confidence is $19/20 = 0.95 = 95\%$

Margin of error $E \approx z_{.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; here $n = 1290$ and polling companies use $\hat{p} = \hat{q} = 0.5$

$E \approx 1.9600\sqrt{\frac{(0.5)(0.5)}{1290}} \approx 0.0273 \approx 2.7\%$

Note: using $\hat{p} = 0.42$ and $\hat{q} = 0.58$, which is also correct in this question since 42% is the only sample percentage quoted in the article, yields $E \approx 0.0269 \approx 2.7\%$.

b) The quoted margin of error (2.7%) applies only to the percentage of investors (42%) who perceive they must incur higher risk to achieve returns offered by foreign investments.

45. 99% CI for p is $\hat{p} \pm z_{.005}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ where $z_{.005} = \text{invNorm}(.995, 0, 1) \approx 2.5758$

99% CI for p is $\frac{680}{800} \pm 2.5758\sqrt{\frac{\frac{680}{800}\left(1-\frac{680}{800}\right)}{800}} \approx 0.85 \pm 0.033$

We are 99% confident that the proportion of all Canadians in favour of the legislation is between 0.817 and 0.883.

46. a) Margin of error $E \approx z_{.025}\sqrt{\frac{\hat{p}\hat{q}}{n}}$; polling companies use $\hat{p} = \hat{q} = 0.5$

For the student poll, $n = 2000$; $E \approx 1.9600\sqrt{\frac{(0.5)(0.5)}{2000}} \approx 0.0219 \approx 2.2\%$

For the adult poll, $n = 1500$; $E \approx 1.9600\sqrt{\frac{(0.5)(0.5)}{1500}} \approx 0.0253 \approx 2.5\%$

b) $E = 1.9600\sqrt{\frac{(0.15)(0.85)}{1500}} \approx 0.0181 \approx 1.8\%$

c) $n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$; here, use $\hat{p} = 0.25,\ \hat{q} = 0.75$; $E = 0.02$

the confidence level = 24 times out of 25 = 24/25 = .96 = 96%,
so, $\alpha = 1 - .96 = .04$ and $z_{\alpha/2} = z_{.02}$ = invNorm(.98, 0, 1) ≈ 2.0537

$n = \frac{(2.0537)^2(.25)(.75)}{(.02)^2} \approx 1977.13$ (round up to nearest integer).

The minimum required sample size is 1978.

d) Neither of the margins of error applies to the percentages for the subgroups. The margin of error depends on the sample size. The subgroups would be represented by a smaller sample (a subset of the 1500 or 2000), and hence, the margins of error would be larger than the margins of error quoted for the overall surveys.