Worked examples on limits and continuity

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Example 1 Use the graph of $y = f(x)$ in Figure 1 on the following page to answer the questions.

1. Find the domain of f.
2. Find $\lim_{x \to 5} f(x)$.
3. Find $\lim_{x \to 2} f(x)$.
4. Find $\lim_{x \to -4^+} f(x)$.
5. Find $\lim_{x \to -7^-} f(x)$.
6. Find $\lim_{x \to \frac{1}{2}} f(x)$.
7. State all values of x in $(-9, 8)$ where f is discontinuous.
8. State all values of x in $(-9, 8)$ where f does not have a limit.
9. List the intervals (as large as possible) where f is continuous.

Solution 1.

1. $(-9, 8) \setminus \{5\}$
2. -1
3. Does not exist
4. -1
5. 2
6. $1/2$

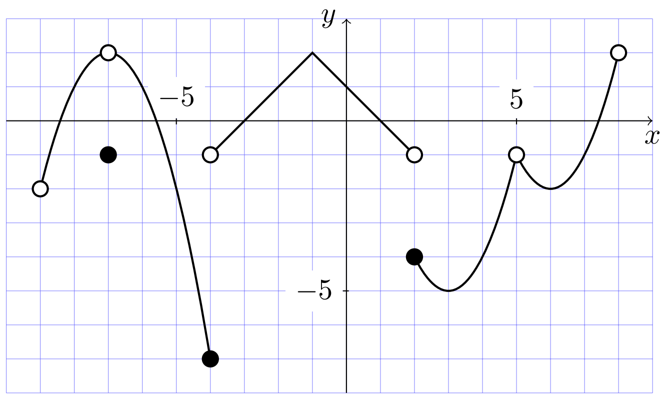

Figure 1: The graph of f

7. $-7, -4, 2, 5$

8. $-4, 2$

9. $(-9, -7) \cup (-7, -4) \cup (-4, 2) \cup (2, 5) \cup (5, 8)$

Example 2 Evaluate the following limits analytically. Use the symbols ∞, $-\infty$, and DNE where appropriate. If the limit does not exist, explain why.

1. $\displaystyle\lim_{x\to 0} \frac{5x^3 - x}{\sin(3x)}$

2. $\displaystyle\lim_{x\to 2^-} \frac{x^2 + x - 6}{|2 - x|}$

3. $\displaystyle\lim_{x\to \infty} \frac{-3x^4 + \cos(x)x^3 - 3}{5000x^4 - 1000\sin^2(x) + 6\sqrt{x}}$

Solution 2.

1. $\lim_{x\to 0} \frac{5x^3-x}{\sin(3x)} = \lim_{x\to 0} \frac{3x}{\sin(3x)} \cdot \frac{5x^2-1}{3} = 1 \cdot \frac{-1}{3} = -\frac{1}{3}$.

2.

$$\begin{aligned}\lim_{x\to 2^-} \frac{x^2 + x - 6}{|2 - x|} &= \lim_{x\to 2^-} \frac{x^2 + x - 6}{2 - x} \\ &= \lim_{x\to 2^-} \frac{(x+3)(x-2)}{2 - x} \\ &= \lim_{x\to 2^-} -(x+3) = -5.\end{aligned}$$

3. The limit is clearly $-\frac{3}{5000}$.

Example 3 Evaluate the following limits analytically. Provide graphs of trigonometric functions when appropriate.

1. $\displaystyle\lim_{x\to 3^-} \frac{2x^2-7x+3}{|x^2+x-12|}$

2. $\displaystyle\lim_{h\to 0} \frac{(h-5)^2-25}{h}$

Solution 3. 1.

$$\begin{aligned}\lim_{x\to 3^-} \frac{2x^2-7x+3}{|x^2+x-12|} &= \lim_{x\to 3^-} \frac{(2x-1)(x-3)}{|(x+4)(x-3)|}\\ &= \lim_{x\to 3^-} \frac{(2x-1)(x-3)}{(x+4)(-1)(x-3)}\\ &= \lim_{x\to 3^-} \frac{2x-1}{(x+4)(-1)} = \frac{5}{-7} = -\frac{5}{7}\end{aligned}$$

2.

$$\begin{aligned}\lim_{h\to 0} \frac{(h-5)^2-25}{h} &= \lim_{h\to 0} \frac{h^2-10h+25-25}{h}\\ &= \lim_{h\to 0} \frac{h^2-10h}{h}\\ &= \lim_{h\to 0} h-10 = -10\end{aligned}$$