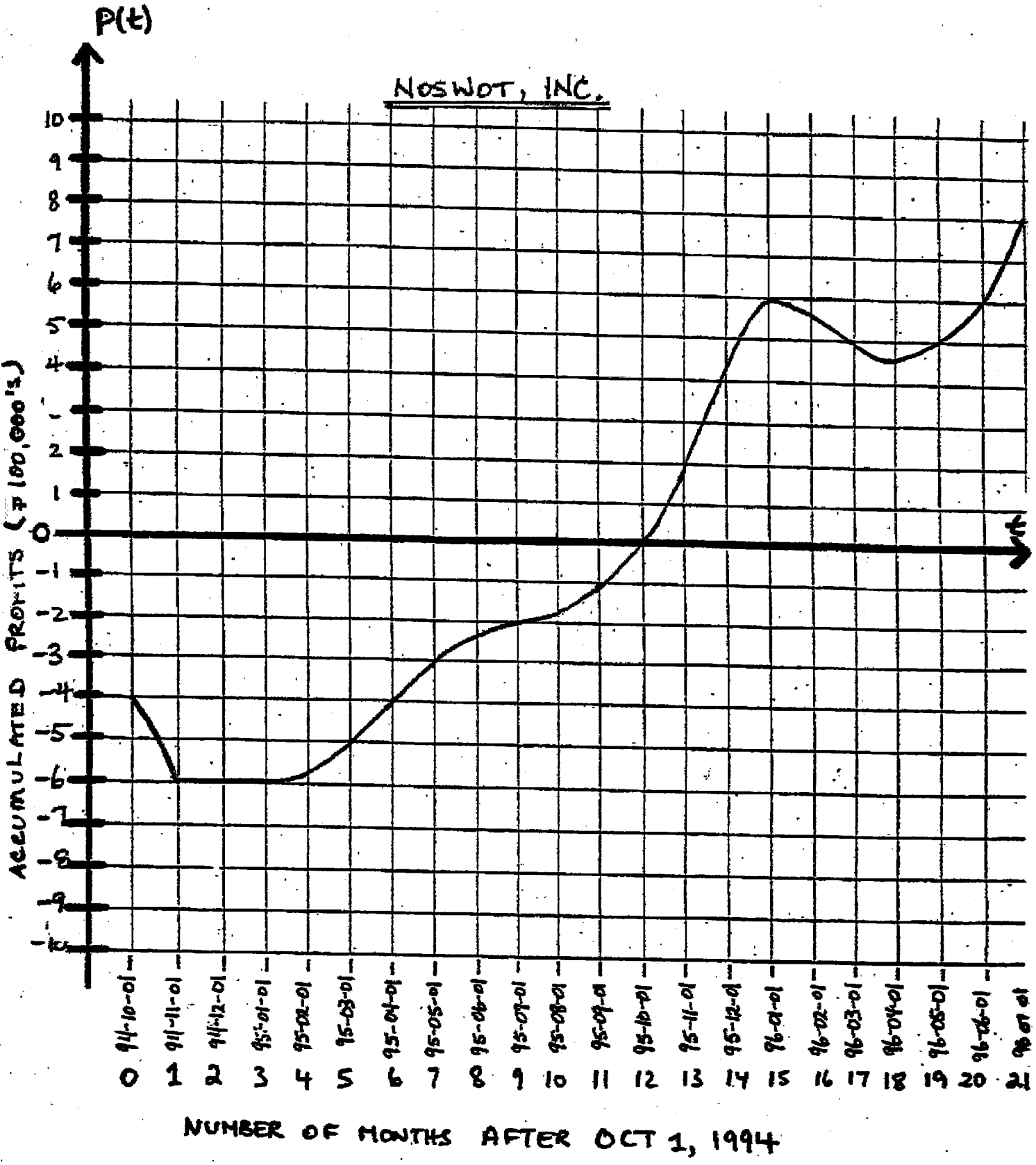
Example: Accumulated profits for a software company is given by the graph below.



- Use the graph to answer the following – be sure to include correct notation and units.

a) NOSWOT, INC. officially started business on Oct. 1/ 94. What was their change in profit during their first month of operation ?

b) What was the average rate of change of profits during this month?

c) What was the average rate of change of profit from Nov. 1/ 94 to April 1/ 95 ?

d) What was the longest period over which profits continually increased ?

e) What was the average rate of change of profit over the period described in (d) ?

f) What was the percentage change in profits during the first two months of 1996 ?

g) What was the average rate of change of profits during the first six months of 1996 ?

Example : Biology: The number N of bacteria in a culture after t days can be modelled by

$$N(t) = 400\left[1 - \frac{3}{(t^2+2)^2}\right]$$

a) What is the initial size of the culture? After 3 days ?

b) What is the total change in the size of the culture over the first 3 days?

c) What is the average rate of change in the size of the culture over the first 4 days?

d) What is the total number of bacteria produced in days # 1 through # 5 ?

e) How many bacteria are produced on day # 3?

f) What is the percent change in total number over the first 3 days ?

Example: The distance in metres of an object from a starting point after t seconds is given by $s(t) = t^2 + 5t + 10$.

a) What is the average velocity of the objcct over the first 3 seconds?

b) What is the average velocity of the object over the first 5 seconds? What appears to be happening?

c) But suppose we want to find the velocity at exactly 5 seconds .We can use this concept to approximate the velocity of the objcct at 5 seconds by finding the average velocity over shorter and shorter time intervals.

Interval	*Average velocity*
$t = 5$ *to* $t = 5.1$ *seconds*	
$t = 5$ *to* $t = 5.01$ *seconds*	
$t = 5$ *to* $t = 5.001$ *seconds*	

The results in the table suggest that the exact velocity at $t = 5$ *seconds* is _____.

This is called the ***instantaneous rate of change of distance with respect to time at*** $t = 5$.

Instantaneous Rate of Change for a function f at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ provided this limit exists.

d) Use this formula to find the instantaneous velocity at $t = 5$ algebraically.

-Putting several concepts together…

Example: A manufacturer can produce MP3 players at a cost of \$20 each. It is estimated that if the players are sold for $\$p$ apiece, consumers will purchase $q = 120 - p$ players each month.

a) Express the manufacturer's profit P as a function of q.

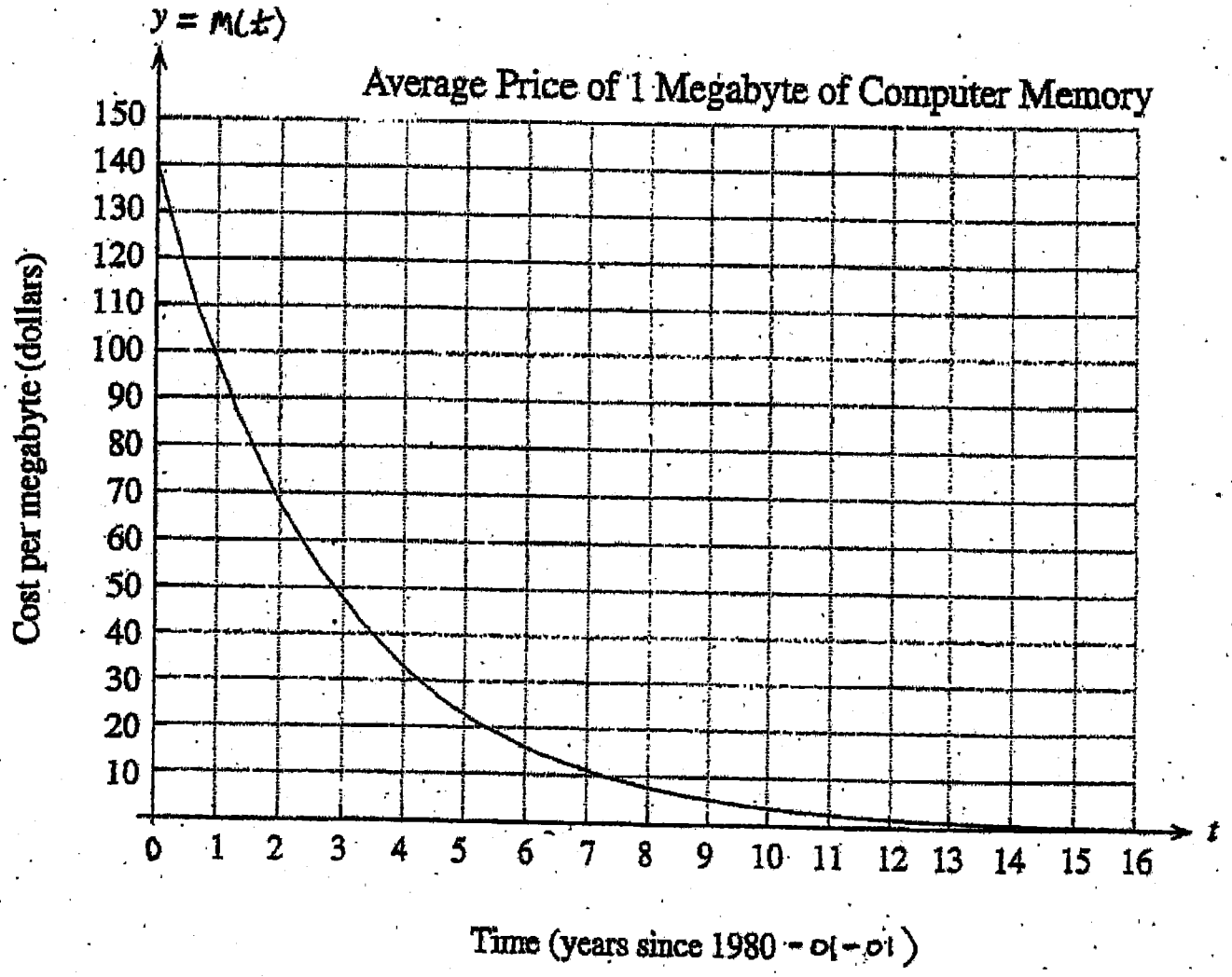
b) What is the average rate of change of profit as the level of production increases from $q = 0$ to $q = 10$?

c) Use the technique on the previous page to determine the rate that the profit is changing when $q = 20$. Is the profit increasing or decreasing at this level of production?

d) What is the break-even level?

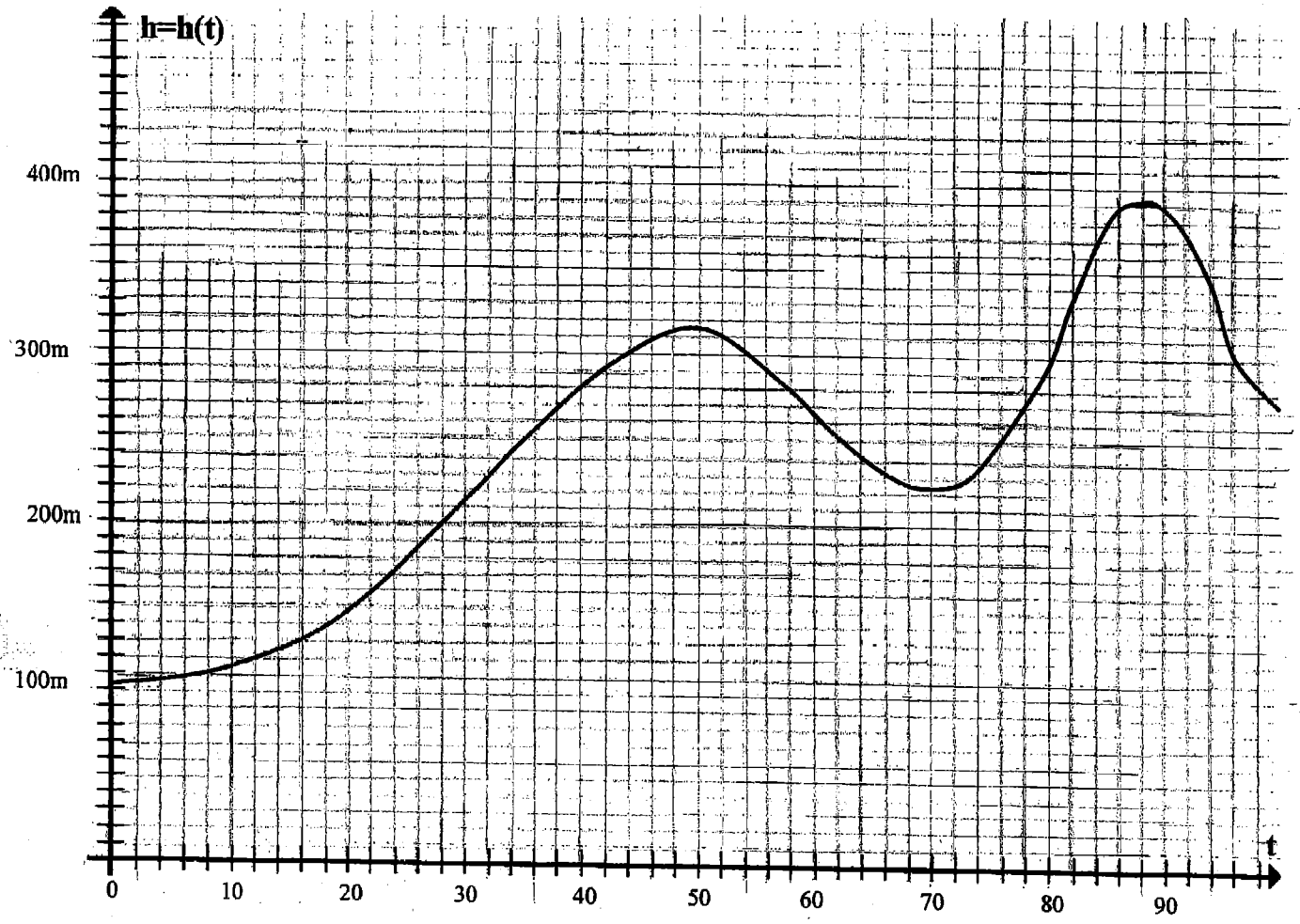
e) At what production level is the Profit a maximum?

Example : The following graph gives the average price of 1 megabyte of computer memory since 1980.



a) How fast was the average price of 1 MB of computer memory changing on Jan. 1 / 83 ?

Example: The following graph shows the elevation of a hot-air balloon during a flight in the Fraser Valley one sunny afternoon. Use the graph to answer the following questions. (Be sure to use appropriate notation wherever relevant).



Time (in minutes) after 2:00 PM

1) What time did the balloon reach its maximum height?

2) What was the change in elevation from 2:10 pm to 2:18 pm?

3) What was the percent change in elevation from 2:10 pm to 2:18 pm ?

4) What was the average rate of change in elevation from 2:10 pm to 2:18 pm?

5) How fast was the balloon rising at 3:05 pm ?

6) What was the rate of change of elevation at 3:20 pm ?

7) What was the *time* and the *rate of ascent* when the balloon was:

a) rising the fastest ?

b) falling the fastest ?

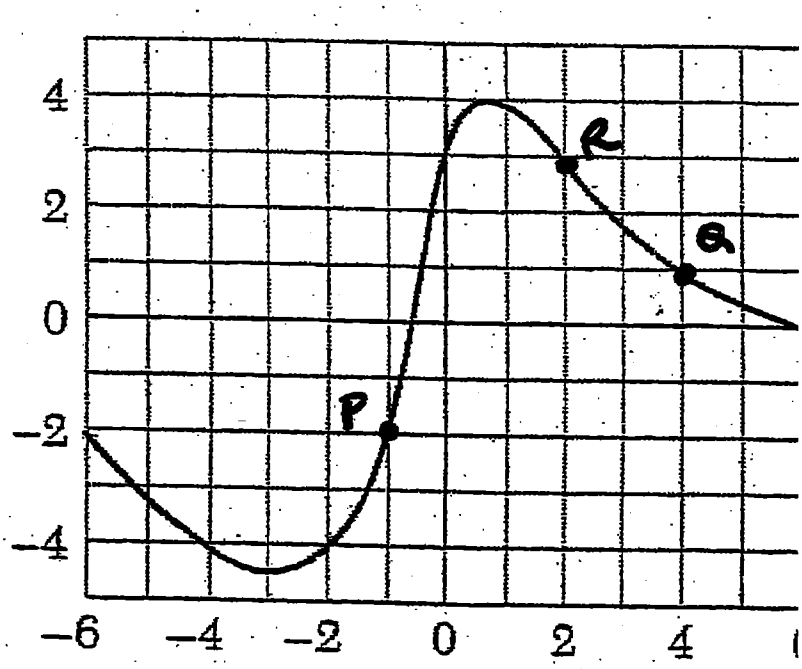
Using Symmetric Difference Quotients:

Example : Approximate the "instantaneous rate of change" of the function f at each indicated point using a "symmetric difference quotient".

a) at point P.

b) at point Q

c) at point R



(Compare your answers with the results from the Example on previous page)

Example : The data below was extracted from the article "VIA Rail Boss Says Service Needs Private Sector Cash" in the Feb. 27, 1998 edition of the Vancouver Sun, p. D4c. It gives the total government funding for VIA Rail.

YEAR	'92	'93	'94	'95	'97	'99
FUNDING: ($ Millions)	385	350	320	295	225	170

Use a symmetric difference quotient to estimate the rate at which government funding for VIA Rail was changing:

a) in 1993

b) in 1997

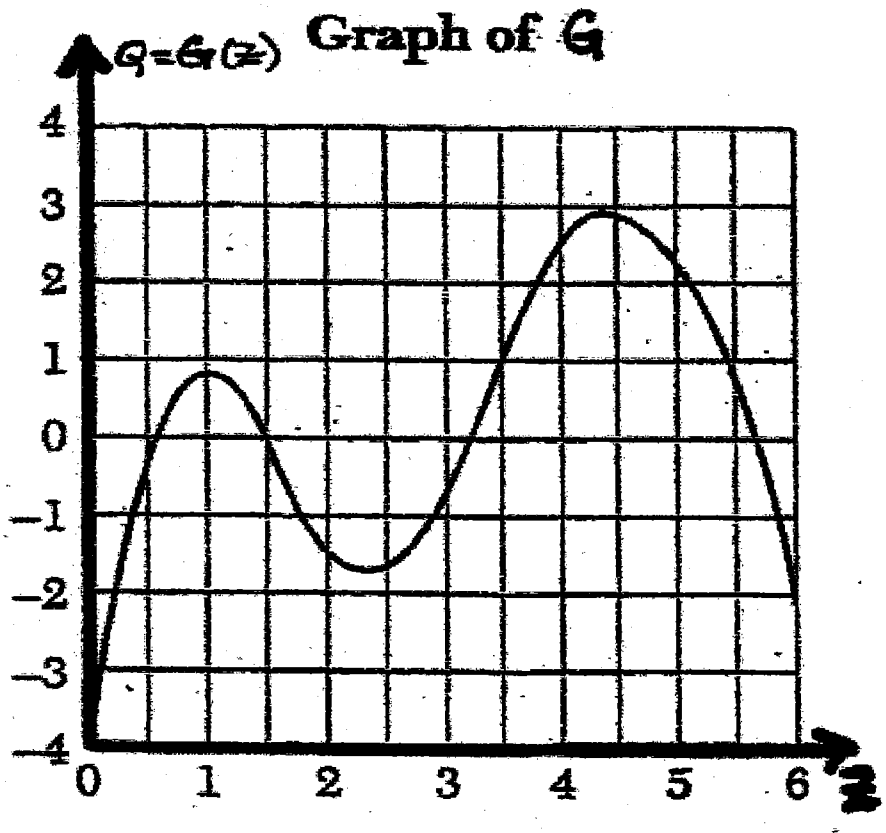
c) in 1996

Example : The spread of a virus is modelled by $V(t)$ where $V(t)$ is the number of people (in hundreds)with the virus and t is the number of weeks since the first case was observed. Carefully interpret the following mathematical statements regarding the virus.

a) $N'(10) = 1.8$

b) $\frac{\Delta N}{\Delta t} = 0.9$ *for* $t = 0$ *to* 5

Example : Use the graph of G to approximate the following:



a) $G(3)$

b) $G'(3)$

c) $\frac{dG}{dz}$ *when* $z = 5$

d) $G'(5)$

e) $\frac{dQ}{dz}$ *when* $z = 5$