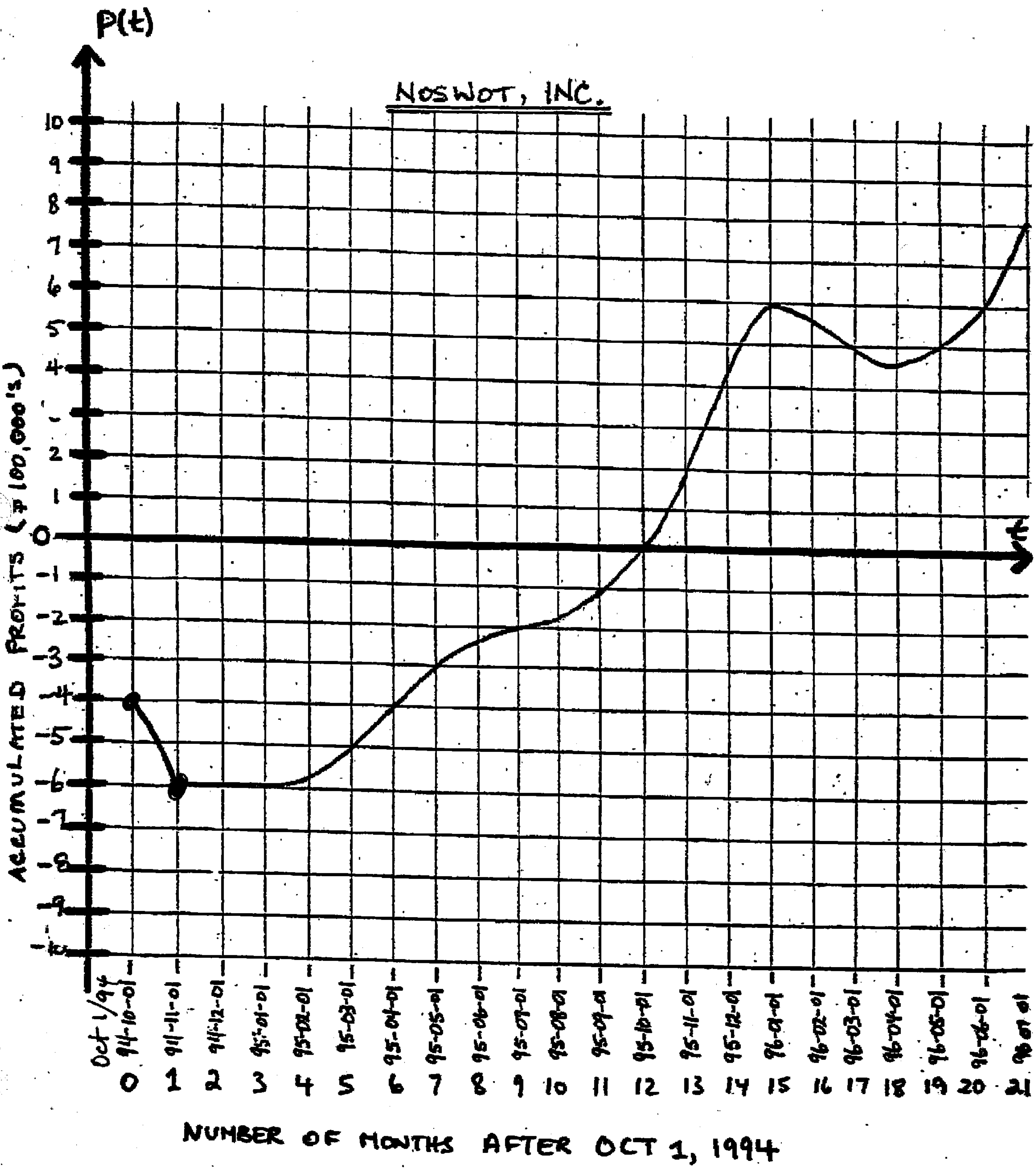
Example: Accumulated profits for a software company is given by the graph below.



- Use the graph to answer the following – be sure to include correct notation and units.

a) NOSWOT, INC. officially started business on Oct. 1/ 94. What was their change in profit during their first month of operation ? $t=0$ to $t=1$

$\Delta P = P(1) - P(0) = -6 - {}^-4 = -2$ hundred thousand \$

$\therefore$ the profit decreased by \$ 200,000.

b) What was the average rate of change of profits during this month?

$$\frac{\Delta P}{\Delta t} = \frac{P(1)-P(0)}{1-0} = \frac{-6 - {}^-4}{1} = \frac{-2 \text{ hundred thousand \$}}{\text{month}}.$$

c) What was the average rate of change of profit from Nov. 1/ 94 to April 1/ 95 ?

$t=1$ $t=6$

$$\frac{\Delta P}{\Delta t} = \frac{P(6)-P(1)}{6-1} = \frac{-4 - {}^-6}{5} = \frac{2}{5} \frac{\text{thousand \$}}{\text{mont}}$$

$\therefore$ \$ 40,000/month

d) What was the longest period over which profits continually increased ?

ie continues to go up from Jan 01/95 to Jan 01/96

$t=3$ $t=15$

e) What was the average rate of change of profit over the period described in (d) ?

$$\frac{\Delta P}{\Delta t} = \frac{P(15)-P(3)}{15-3} = \frac{6 - {}^-6}{12} = \frac{12}{12}$$

$\therefore$ \$ 100,000/month

Jan 01 - Mar 01

f) What was the percentage change in profits during the first two months of 1996 ? $t=15$ to $t=17$

$$\% \Delta = \frac{\text{total change}}{\text{original value}} \times 100\% = \frac{\Delta P}{P(t_1)} \times 100\% = \frac{5-6}{6} \cdot 100\%$$

$\approx \boxed{-16.7\%}$

g) What was the average rate of change of profits during the first six months of 1996 ?

Jan 01 to July 1

$t=15$ $t=21$

$$\frac{\Delta P}{\Delta t} = \frac{P(21)-P(15)}{21-15} = \frac{8-6}{6} = \frac{2}{6} = \frac{1}{3}$$

ie $\boxed{\text{\$ 33,333/month}}$

Example : **Biology:** The number N of bacteria in a culture after t days can be modelled by

$$N(t) = 400\left[1 - \frac{3}{(t^2+2)^2}\right] \rightarrow \text{enter into calculator } y_1$$

a) What is the initial size of the culture? After 3 days ?

use TABLE

$N(0) = 100$ Bacteria $\quad N(3) \approx 390$ bacteria

b) What is the total change in the size of the culture over the first 3 days?

$t=0$ to $t=3$

$$\Delta N = N(3) - N(0)$$
$$= y_1(3) - y_1(0) = \boxed{290} \text{ bacteria}$$

y_1 $\boxed{\text{VARS}}$

c) What is the average rate of change in the size of the culture over the first 4 days?

$$\frac{\Delta N}{\Delta t} = \frac{y_1(4) - y_1(0)}{4-0} \approx 74 \text{ bacteria/day}$$

d) What is the total number of bacteria produced in days # 1 through # 5 ?

ie ΔP from $t=0$ to $t=5$

$\Delta P = N(5) - N(0) \approx 298$ bacteria

e) How many bacteria are produced on day # 3?

ie change from day 2 to day 3 #

$\Delta P = N(3) - N(2) \approx 23$ bacteria

f) What is the percent change in total number over the first 3 days ?

$$\% \text{ change} = \frac{\text{Total change}}{\text{original}} \times 100\,\%$$

$$= \frac{N(3)-N(0)}{N(0)} \cdot 100\,\%$$

$$\approx \boxed{290.08\,\%}$$

Quiz # 4 to here.

Example: The distance in metres of an object from a starting point after t seconds is given by $s(t) = t^2 + 5t + 10$. → enter into y_1

a) What is the average velocity of the object over the first 3 seconds? $t=0$ to $t=3$

$$\frac{\Delta s}{\Delta t} = \frac{s(3)-s(0)}{3-0} = \frac{y_1(3)-y_1(0)}{3} = \frac{24}{3} = 8 \text{ m/sec}$$

b) What is the average velocity of the object over the first 5 seconds? What appears to be happening?

$$\frac{\Delta s}{\Delta t} = \frac{s(5)-s(0)}{5-0} = \frac{50}{5} = 10 \text{ m/sec}$$ Speeding up.

c) But suppose we want to find the velocity at exactly 5 seconds. We can use this concept to approximate the velocity of the object at 5 seconds by finding the average velocity over shorter and shorter time intervals.

Interval	*Average velocity*
$t = 5$ to $t = 5.1$ seconds	$\frac{\Delta s}{\Delta t} = \frac{s(5.1)-s(5)}{5.1-5} = \frac{1.51}{.1} = 15.1$ m/sec
$t = 5$ to $t = 5.01$ seconds	$\frac{\Delta s}{\Delta t} = \frac{s(5.01)-s(5)}{5.01-5} = \frac{.1501}{.01} = 15.01$ m/sec
$t = 5$ to $t = 5.001$ seconds	$\frac{\Delta s}{\Delta t} = \frac{s(5.001)-s(5)}{5.001-5} = \frac{.015001}{.001} = 15.001$ m/sec

The results in the table suggest that the exact velocity at $t = 5$ *seconds* is 15 m/sec.

This is called the ***instantaneous rate of change of distance with respect to time at t = 5.***

Must KNOW

Instantaneous Rate of Change for a function f at $x = a$ is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ provided this limit exists.

s
$a = 5$
$h = \Delta t$

d) Use this formula to find the instantaneous velocity at $t = 5$ algebraically.

$s(t) = t^2 + 5t + 10$

$$\lim_{h\to 0} \frac{s(5+h) - s(5)}{h}$$

$$= \lim_{h\to 0} \frac{[(5+h)^2 + 5(5+h) + 10] - [5^2 + 5(5) + 10]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{25} + 10h + h^2 + \cancel{25} + 5h + \cancel{10} - \cancel{60}}{h}$$

$$= \lim_{h\to 0} \frac{15h + h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}(15+h)}{\cancel{h}} = 15 + 0 = \boxed{15 \text{ m/sec}}$$

-Putting several concepts together...

Example: A manufacturer can produce MP3 players at a cost of \$20 each. It is estimated that if the players are sold for $\$p$ apiece, consumers will purchase $q = 120 - p$ players each month.

$\to p = 120 - q$

a) Express the manufacturer's profit P as a function of q.

$$P = \text{Revenue} - \text{Cost}$$
$$= (120q - q^2) - (20q)$$
$$\boxed{P(q) = 100q - q^2}$$

Revenue = price × quantity
$= p \cdot q$
$= (120 - q)q$

Cost = (price per unit)(# units)

b) What is the average rate of change of profit as the level of production increases from $q = 0$ to $q = 10$?

$$\frac{\Delta P}{\Delta q}$$

c) Use the technique on the previous page to determine the rate that the profit is changing when $q = 20$. Is the profit increasing or decreasing at this level of production?

"instantaneous rate of change" "a"

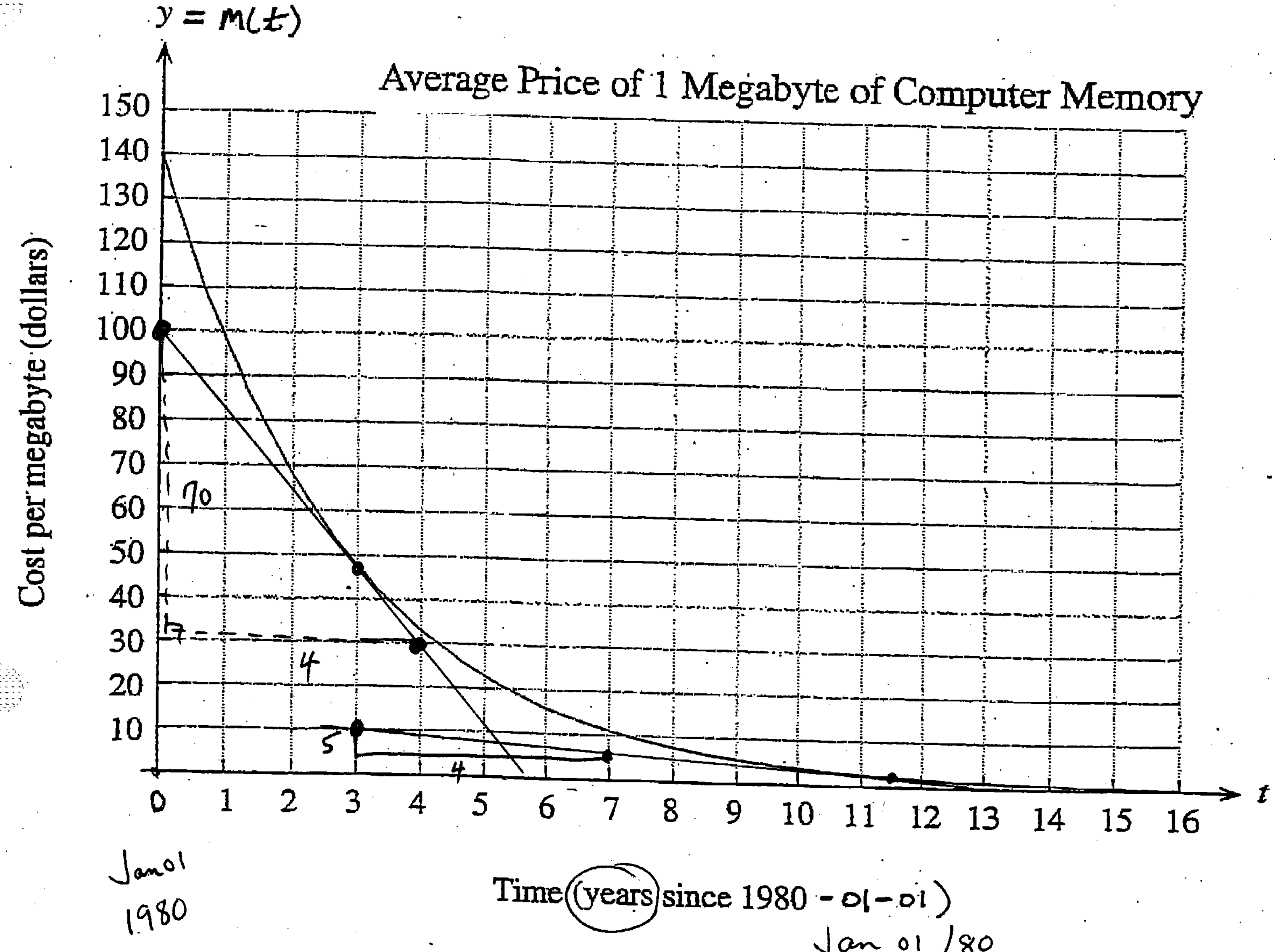
$$\lim_{h \to 0} \frac{P(20+h) - P(20)}{h}$$

d) What is the break-even level?

$R = C$ OR $P = 0$

e) At what production level is the Profit a maximum?

Example: The following graph gives the average price of 1 megabyte of computer memory since 1980.



a) How fast was the average price of 1 MB of computer memory changing on $\boxed{\text{Jan. 1 / 83 ?}}$

1 point ∴ instantaneous rate of change

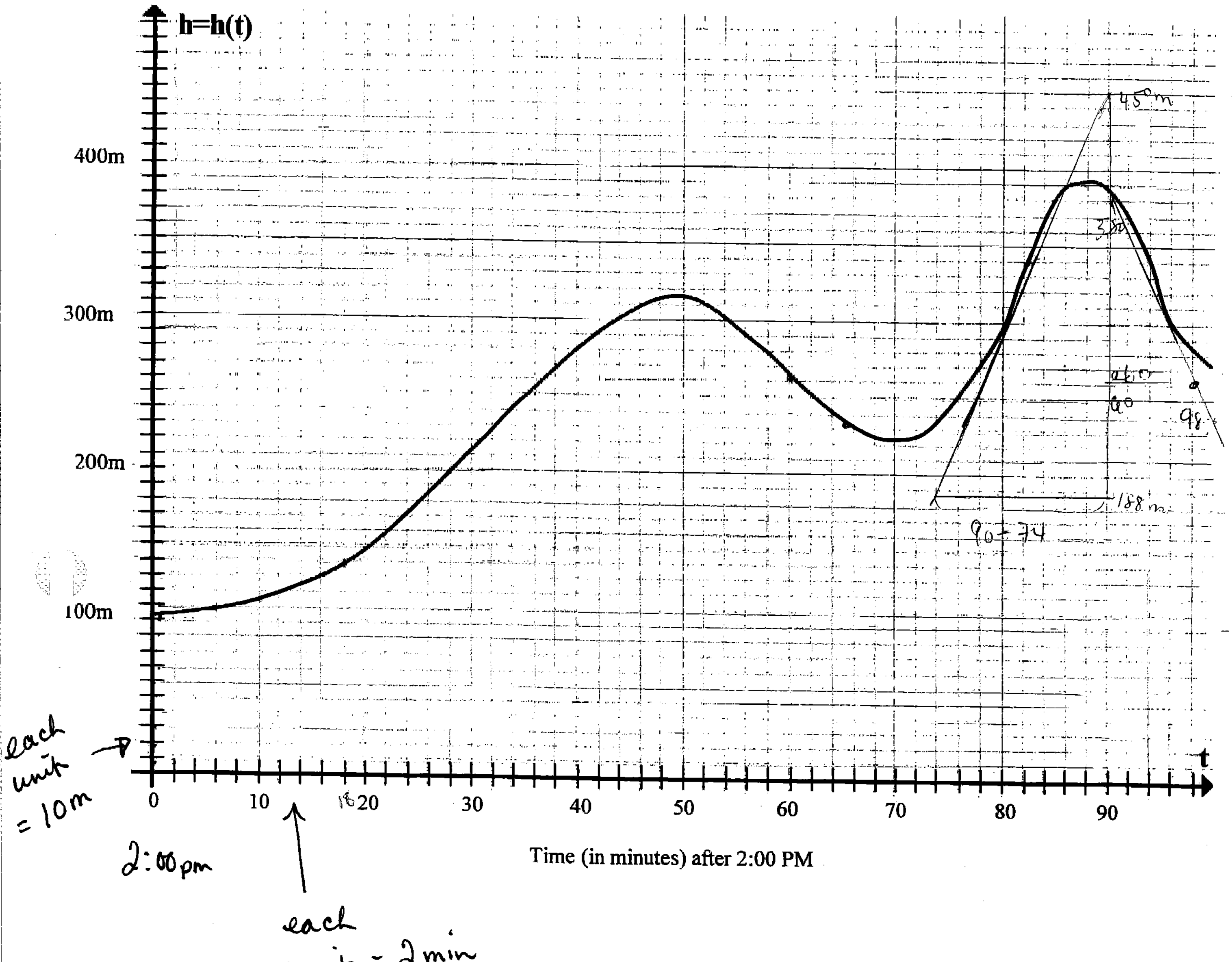
$$\left.\frac{dm}{dt}\right|_{t=3} \approx \frac{-70}{4} \approx \frac{-35}{2} \approx -17.50 \ \frac{\$ \text{ per Mb}}{\text{year}} \qquad t=3$$

b) How fast was the average price of 1 MB of computer memory changing on $\boxed{\text{July 1 / 91 ?}}$

$t = 11.5$

$$\left.\frac{dm}{dt}\right|_{t=11.5} \approx \frac{-5}{4} = -1.25 \ \frac{\$ \text{ per MB}}{\text{year}}$$

Example: The following graph shows the elevation of a hot-air balloon during a flight in the Fraser Valley one sunny afternoon. Use the graph to answer the following questions. (Be sure to use appropriate notation wherever relevant).



1) What time did the balloon reach its maximum height?

Find t at max height

$t =$ $\therefore$ time is

2) What was the change in elevation from 2:10 pm to 2:18 pm?

Find Δh from $t = 10$ to $t = 18$

$\Delta h = h(18) - h(10) =$

3) What was the percent change in elevation from 2:10 pm to 2:18 pm ?

$$\% \Delta = \frac{\Delta h}{h_1} \cdot 100\% = \frac{h(18) - h(10)}{h(10)} \cdot 100\%$$

4) What was the average rate of change in elevation from 2:10 pm to 2:18 pm?

$$\frac{h(18) - h(10)}{8} \text{ m/minute}$$

5) How fast was the balloon rising at 3:05 pm ?

Find instantaneous rate of change @ $t = 65$

$$\left.\frac{dh}{dt}\right|_{t=65}$$

6) What was the rate of change of elevation at 3:20 pm ?

$$\left.\frac{dh}{dt}\right|_{t=80}$$

7) What was the *time* and the *rate of ascent* when the balloon was:

a) rising the fastest ? Steepest slope at 3:20 pm

at $\frac{(450 - 188)\text{m}}{(90 - 74)\text{ minutes}}$

b) falling the fastest ? 96 minutes after 2pm 3:36pm

$$-\frac{(380 - 260)}{98 - 90} \text{ m/minute}$$

Example : A manufacturer of a certain product has determine that the total cost of producing x units of this product can bc represented by $C(x)$ where $C(x)$ is in \$. Carefully interpret the following mathematical statements regarding the product. in non-mathematical terms.

$\frac{dc}{dx}$

a) $C'(50) = 28.72$ When 50 units are produced, the cost is changing at a rate of 28.72 \$/unit

b) $\frac{dC}{dx} = -25.09$ *when* $x = 124$

When 124 units are product, the cost is decreasing at a rate 25.09 \$/unit

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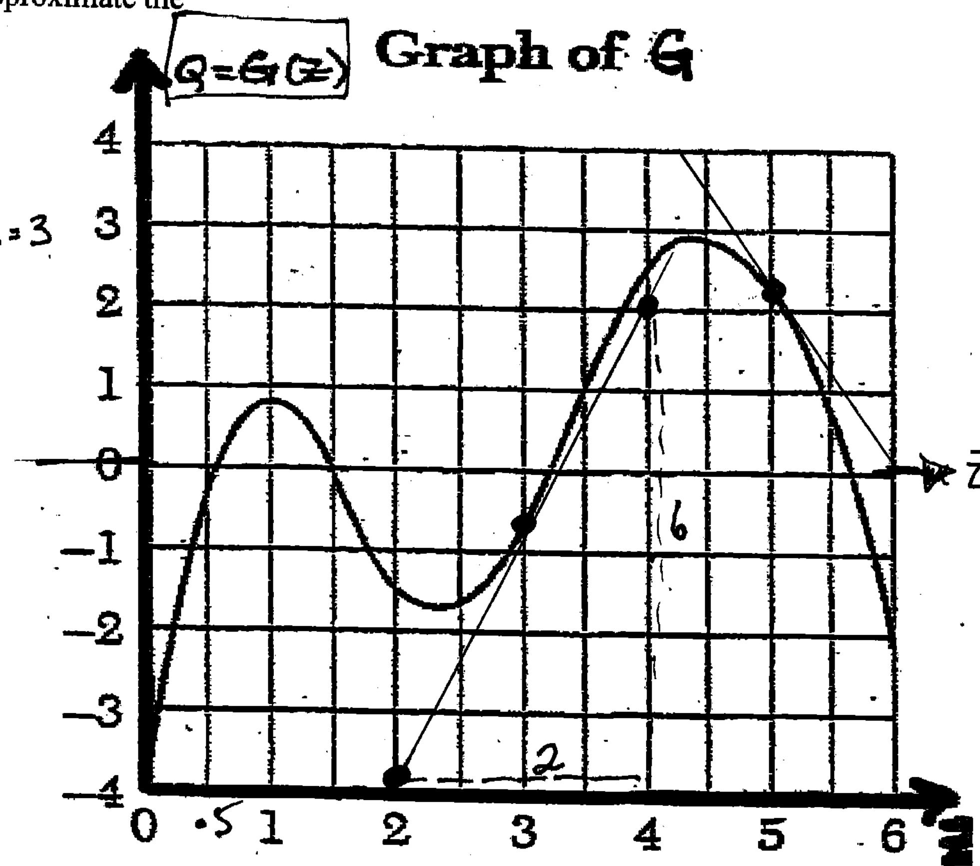
Example : Use the graph of G to approximate the following:



a) $G(3)$ output pt on graph

$G(3) \approx -.75$

b) $G'(3)$ tgt slope @ z=3

$G'(3) \approx 6/2 = 3$

c) $\frac{dG}{dz}$ *when* $z=5$

tgt slope @ z=5

d) $G'(5)$ – same as (c)

e) $\frac{dQ}{dz}$ *when* $z=5$

– same as (c) & (d)