Worked examples on Microsoft's revenue and gas prices

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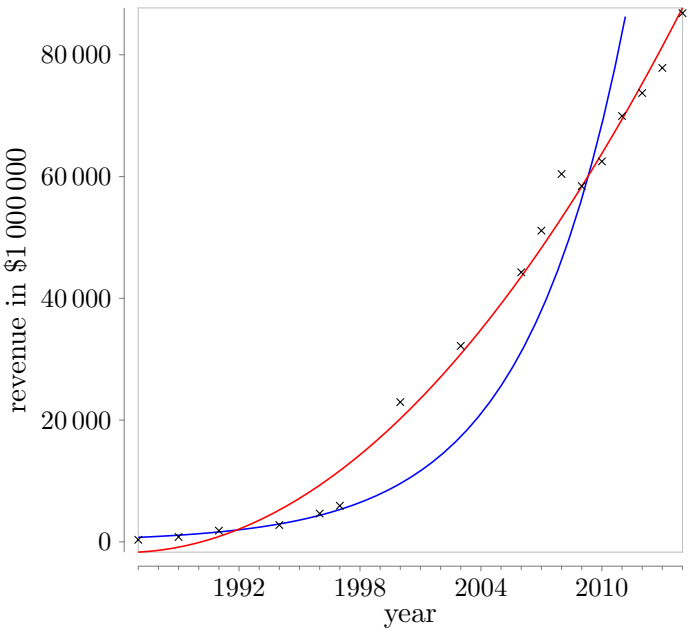
Example The table and graph show Microsoft Corporation's annual revenue (in millions of dollars for the year ending on June 30th of each year).[1]

year	revenue
1987	346
1989	804
1991	1 843
1994	2 759
1996	4 649
1997	5 937
2000	22 956
2003	32 187
2006	44 282
2007	51 122
2008	60 420
2009	58 437
2010	62 484
2011	69 943
2012	73 723
2013	77 849
2014	86 833



1. Select the two most reasonable types of models for the given data. Then find the best-fit model for each type.

[1]Source: http://www.microsoft.com

Solution Subtract 1987 from each year[2] and store them in L_1. Store the revenues in L_2.

(a) Exponential regression (in blue in the graph above):

$$y = 740.9589140(1.217590863)^x$$

(b) Quadratic regression (in red):

$$y = 116.0816344x^2 + 176.4345062x - 1687.947674$$

2. Use each model to predict in which year revenues will reach \$25 billion.

Solution (a) Solving $740.9589140(1.217590863)^x = 25000$ with your graphing calculator yields that $x = 17.87276269$, so with the exponential model revenue will reach \$25 billion in year 2005.

(b) Similarly, if $116.0816344x^2 + 176.4345062x - 1687.947674 = 25000$, then $x = 14.42174920$, so the quadratic model gives year 2001.

Note that the quadratic model agrees better with the data.

3. Use each model to predict the annual revenue for (the year ending on June 30th)

(a) 1990

Solution \$1338 million with the exponential model and a *negative* revenue of \$114 million with the quadratic model. While revenues *can* be negative, the exponential model seems better.

(b) 2005

Solution \$25 634 million with the exponential model and \$39 098 million with the quadratic model. Here the quadratic model agrees better with the data.

4. Which model do you think fits the data better?

Solution The exponential model fits the early data best, but from around year 2000 then quadratic model fits best.

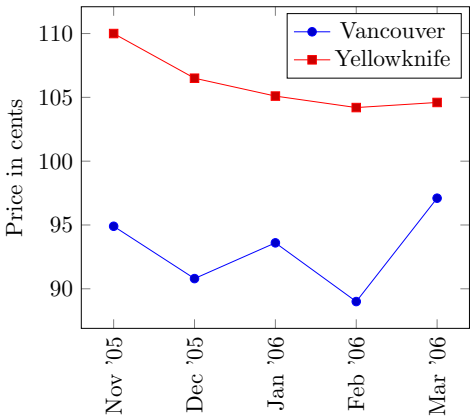
[2]If you do not shift the years, your calculator will run into trouble with overflow.

5. Discuss the relative merits of the two models for extrapolating backwards to years prior to 1987.

   Solution The quadratic model is symmetric around some time in 1986, which is not realistic. The exponential model levels off towards zero, which is more plausible.

Example The price in cents for one litre of regular unleaded gas in Vancouver and Yellowknife is given below.

date	Vancouver	Yellowknife
Nov '05	94.9	110.0
Dec '05	90.8	106.5
Jan '06	93.6	105.1
Feb '06	89.0	104.2
Mar '06	97.1	104.6



1. Based on the data above, choose the most appropriate model and use your graphing calculator to find the best-fit model of that type.

   (a) For Vancouver

   (b) For Yellowknife

   Solution Store the date as the number of months since November 2005 in L_1, so $L_1 = \{0, 1, 2, 3, 4\}$. Store the gas prices in L_2 and L_3.

   (a) In Vancouver use a quartic regression model since the graph looks like a W:

   $$y_{\text{van}} = 1.433x^4 - 10.983x^3 + 26.367x^2 - 20.917x + 94.9$$

   (If you based time on October 2005, so $L_1 = \{1, 2, 3, 4, 5\}$, the function is of course slightly different: $y_{\text{van}} = 1.433x^4 - 16.717x^3 + 67.917x^2 - 112.333x + 154.6$.)

(b) In Yellowknife you could try an exponential model:

$$y_{\text{yke}} = 108.69(0.987824)^x$$

However, with this model, the gas price in Yellowknife would eventually become arbitrarily small. Moreover, last data point increases slightly over the previous one, so a quadratic regression model might be better:

$$y_{\text{ykq}} = 0.5928571429x^2 - 3.681428571x + 109.8857143$$

(If you based time in October, $y_{\text{yke}} = 110.03(0.987824)^x$ and $y_{\text{ykq}} = 0.5928571429x^2 - 4.867142857x + 114.16$.)

2. Use your models to predict

   (a) Vancouver gas prices in September 2005.

   (b) Yellowknife gas prices in September 2006.

Solution (a) If $x = -2$, then $y_{\text{van}} = 353$, which is not very realistic.

(b) If $x = 10$, then $y_{\text{yke}} = 96.2$ and $y_{\text{ykq}} = 132.4$.

(If you based time in October, you must of course use $x = -1$ and $x = 11$.)