4. Symmetry. $x = -1, x = 1, y = x^{1/3}$.

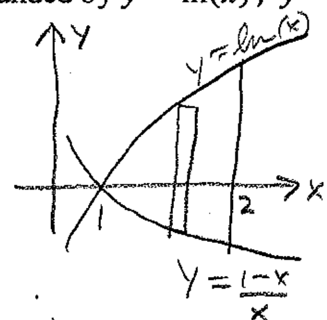
Horizontal rectangles are used when x is a function of y. If $f(y) \geq g(y)\ for\ all\ y\ in\ [c, d]$ then

$$Area = \int_c^d (f(y) - g(y))dy$$

Right minus Left

5. $3y - x = 6,\ \ x + y = -2,\ \ x + y^2 = 4.$

6. Give integral expressions for the area of the region bounded by $y = \ln(x)$, $y = \frac{1-x}{x}, and\ x = 2$



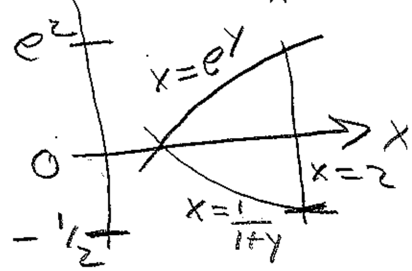
(a) using x as the variable of integration;

$$A = \int_1^2 \left(\ln(x) - \frac{1-x}{x}\right)dx$$

(b) using y as the variable of integration.

$xy + x = 1$

$x = \frac{1}{1+y}$



$$A = \int_{-1/2}^{0} \left(2 - \frac{1}{1+y}\right)dy + \int_0^{e^2} (2 - e^y)dy$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.

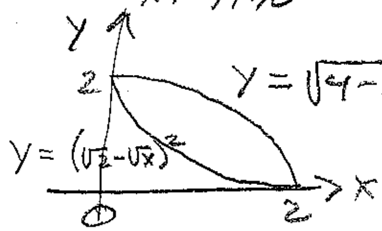
$y^{1/2} = \sqrt{2} - \sqrt{x}$

$y = (\sqrt{2} - \sqrt{x})^2$

Area exercises

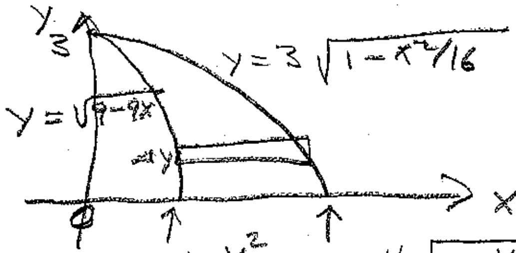
1. $x^{1/2} + y^{1/2} = \sqrt{2}$, $x^2 + y^2 = 4$.

$x \geq 0, y \geq 0$ $\quad y = \sqrt{4 - x^2}$



$$A = \int_0^2 \left(\sqrt{4-x^2} - (\sqrt{2}-\sqrt{x})^2\right) dx$$

2. The region in the first quadrant bounded by $\frac{x^2}{16} + \frac{y^2}{9} = 1$, $x = 1 - \frac{y^2}{9}$.



$\frac{y^2}{9} = 1 - \frac{x^2}{16}$

$y = 3\sqrt{1 - x^2/16}$

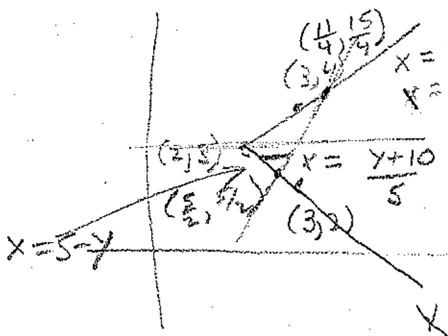
$9x = 9 - y^2$

$y^2 = 9 - 9x$

$y = \sqrt{9-9x}$

$$A = \int_0^3 \left(4\sqrt{1-\frac{y^2}{9}} - \left(1 - \frac{y^2}{9}\right)\right) dy$$

3. $x = |y-3| + 2$, $y = 5x - 10$.



$x = y - 3 + 2$

$x = y - 1$, $y = x + 1$

$y = 5x - 10 = x + 1$

$4x = 11$

$x = 11/4$

$y = x + 1 = 15/4$

$x = 3 - y + 2$

$x = 5 - y$, $y = 5 - x$

$y = 5x - 10 = 5 - x$

$6x = 15$

$x = 15/6 = 5/2$ $\quad (\frac{5}{2}, \frac{5}{2})$

$y = 5 - \frac{5}{2} = 5/2$

$$A = \int_{y=5/2}^{3} \left(\frac{y+10}{5} - (5-y)\right) dy + \int_{y=3}^{15/4} \left(\frac{y+10}{5} - (y-1)\right) dy$$

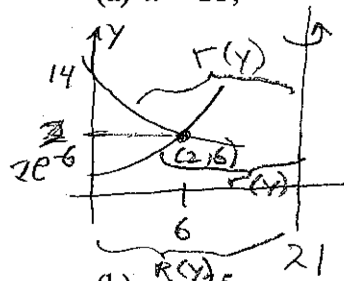
Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.

$\frac{y}{2} = e^{x-6}$, $x-6 = \ln\left(\frac{y}{2}\right)$, $x = 6 + \ln\left(\frac{y}{2}\right)$

4. The region *A* bounded by the *y-axis*, $y = 2e^{x-6}$, $y = \dfrac{14}{x+1}$. Sketch *A*. $x+1 = 14/y$, $x = \frac{14}{y} - 1$

Set up integrals (but do not evaluate) for the volume of the solid obtained by rotating *A* about the following:

(a) $x = 21$;



From $y = 2e^{-6}$ to $y = 2$

$R(y) = 21$

$r(y) = 21 - (6 + \ln(\frac{y}{2})) = 15 - \ln(\frac{y}{2})$

From $y = 2$ to $y = 14$

$R(y) = 21$

$r(y) = 21 - (\frac{14}{y} - 1) = 22 - 14/y$

$$V = \pi \int_{2e^{-6}}^{2} \left\{ 21^2 - \left(15 - \ln\left(\frac{y}{2}\right)\right)^2 \right\} dy + \pi \int_{2}^{14} \left\{ 21^2 - \left(22 - \frac{14}{y}\right)^2 \right\} dy$$

(b) $y = 15$.

Volumes of known cross sectional area

If a solid *S* has a base region *A* and the cross-sections perpendicular to the *x-axis* are *A(x)* then small portions of the solid have volume approximately $A(x_i)\Delta x$ after partitioning the *x-axis* in the usual way. Consequently the volume of the solid can be approximated as

$$V(S) \approx \sum_{i=1}^{n} A(x_i)\Delta x$$

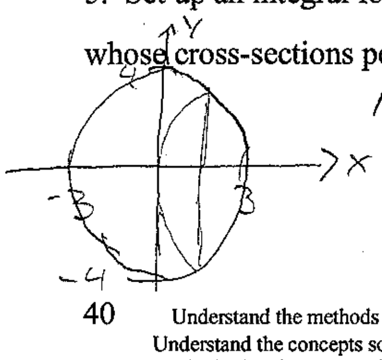
and at the limit the volume is

$$V(S) = \int_a^b A(x)dx.$$

A similar formula applies if the cross-sections are parallel to the *y-axis*.

5. Set up an integral for the volume of a solid whose base is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ and whose cross-sections perpendicular to the *x-axis* are semicircles.

$y = \pm 4\sqrt{1 - \frac{x^2}{9}}$



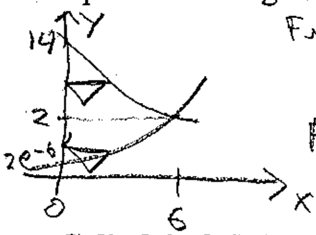
$A = \frac{1}{2}\pi r^2$, $r = 4\sqrt{1 - \frac{x^2}{9}}$

$A(x) = \frac{1}{2}\pi\left(4\sqrt{1 - \frac{x^2}{9}}\right)^2$

$= 8\pi\left(1 - \frac{x^2}{9}\right)$, $V = \int_{-3}^{3} 8\pi\left(1 - \frac{x^2}{9}\right) dx$

$$A_\Delta = \frac{b^2\sqrt{3}}{4}$$

6. Set up an integral for the volume of a solid whose base is the region A bounded by the *y-axis*, $y = 2e^{x-6}$, $y = \dfrac{14}{x+1}$ and whose cross-sections perpendicular to the *y-axis* are equilateral triangles.



From $y = 2e^{-6}$ to $y = 2$

$b = 6 + \ln\left(\frac{y}{2}\right)$

From $y = 2$ to $y = 14$

$b = \frac{14}{y} - 1$

$$V = \int_{2e^{-6}}^{2} \frac{\left(6+\ln\left(\frac{y}{2}\right)\right)^2}{4}\sqrt{3}\,dy + \int_{2}^{14} \frac{\left(\frac{14}{y}-1\right)^2}{4}\sqrt{3}\,dy$$

Cylindrical shells

If a region is rotated about a vertical line to create a solid S, the volume can be calculated by taking a cut parallel to the line, of height $h(x)$ and at distance $p(x)$ from the line. In this case the volume is

$$V(S) = 2\pi\int_a^b p(x)h(x)dx.$$

7. Use cylindrical shells to set up an integral for the volume of the solid obtained by rotating the region bounded by $y = x^3$ and $y=x^2$, $0 \le x \le 1$ about the following:

a) $y = 3$;

b) $x = -12$.

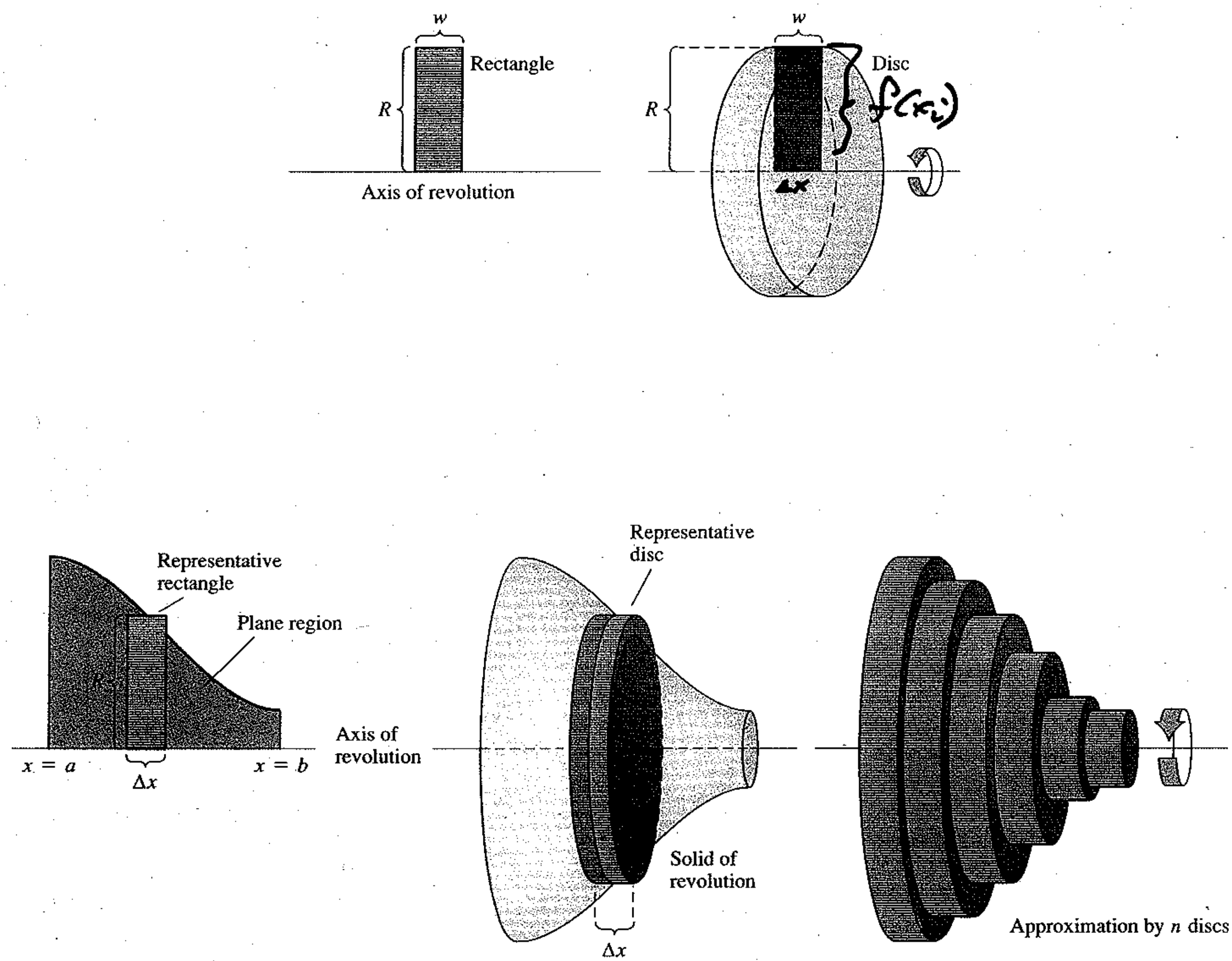


FIGURE 33 The Disc Method

Calculus, Fourth Edition, Larson/Hostetler/Edwards
Calculus, Alternate Fourth Edition, Larson/Hostetler

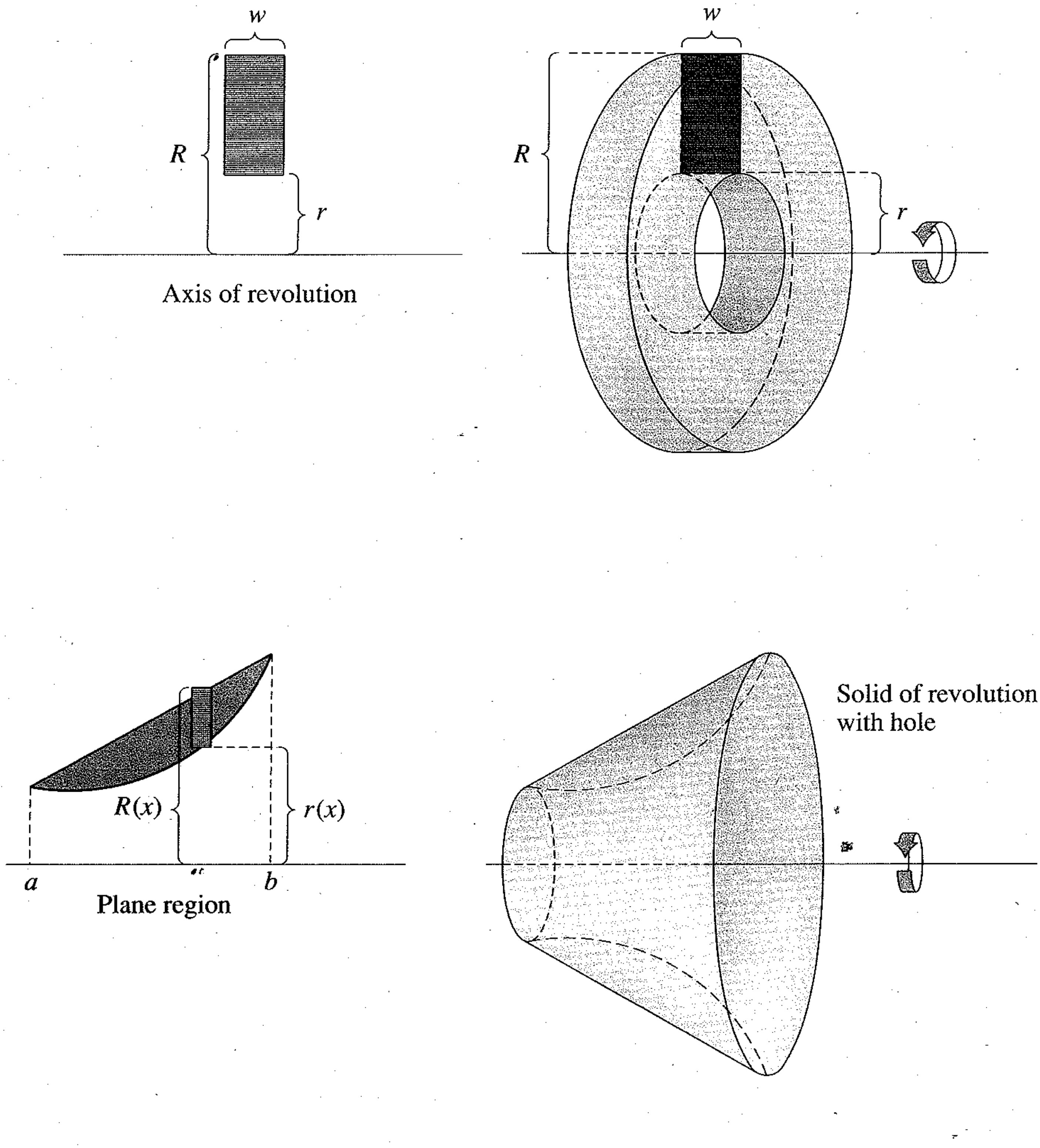
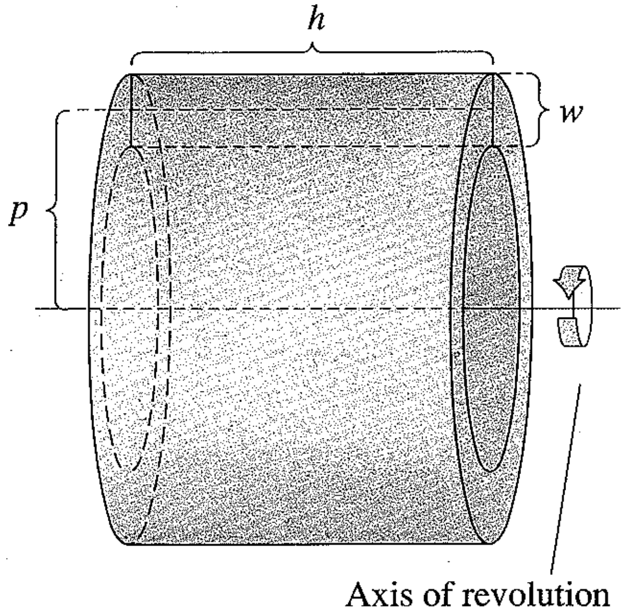


FIGURE 34 The Washer Method

Calculus, Fourth Edition, Larson/Hostetler/Edwards
Calculus, Alternate Fourth Edition, Larson/Hostetler



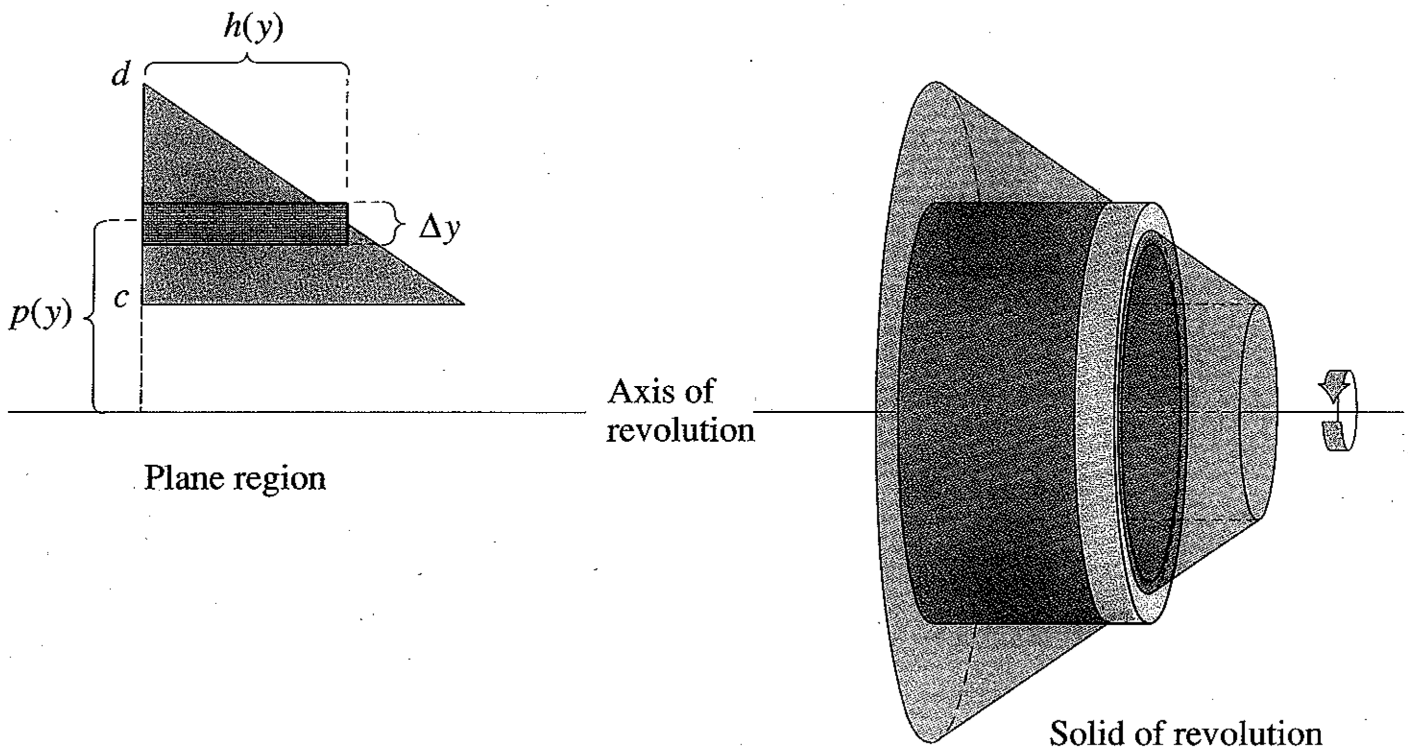
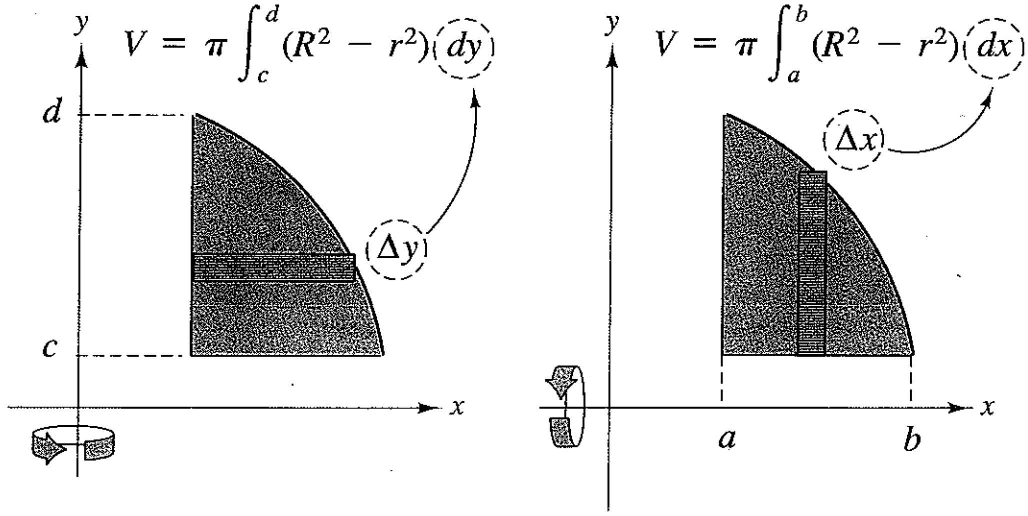


FIGURE 37 The Shell Method

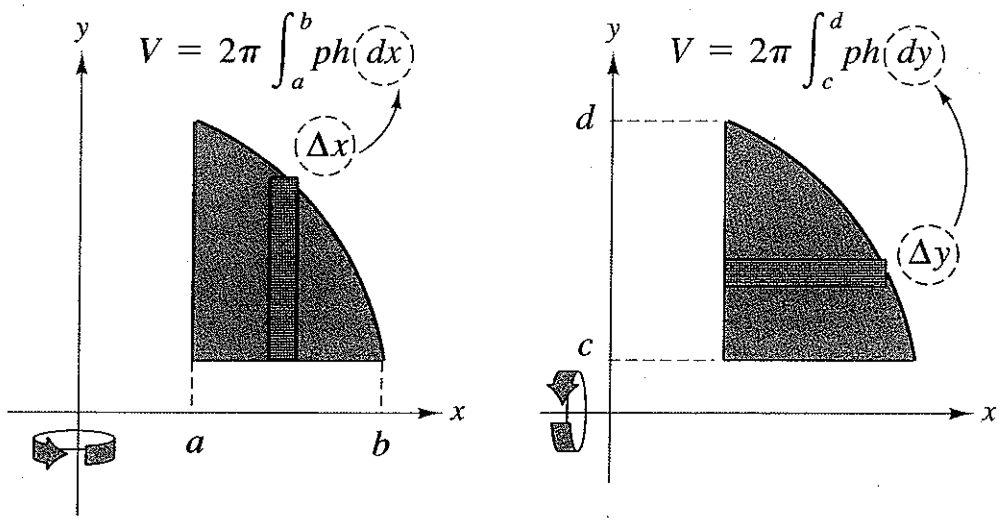
Calculus, Fourth Edition, Larson/Hostetler/Edwards
Calculus, Alternate Fourth Edition, Larson/Hostetler



(a) Vertical axis of revolution

Horizontal axis of revolution

Disc Method: Representative rectangle is perpendicular to the axis of revolution.



(b) Vertical axis of revolution

Horizontal axis of revolution

Shell Method: Representative rectangle is parallel to the axis of revolution.

FIGURE 39 Comparison of Disc and Shell Methods

Calculus, Fourth Edition, Larson/Hostetler/Edwards
Calculus, Alternate Fourth Edition, Larson/Hostetler