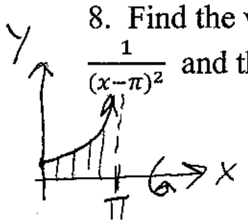
8. Find the volume of the solid obtained by rotating the region bounded by $f(x) = \frac{1}{(x-\pi)^2}$ and the x-axis on the interval $[0,\pi)$ about the (a) x-axis;



Washers, $V = \pi \int_0^{\pi} \left(\frac{1}{(x-\pi)^2}\right)^2 dx$

$= \pi \lim_{t \to \pi^-} \left(\frac{(x-\pi)^{-3}}{-3} \Big|_0^t \right)$

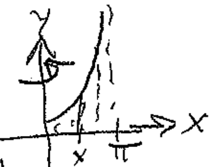
$= -\frac{\pi}{3} \lim_{t \to \pi^-} \left(\frac{1}{(t-\pi)^3} - \frac{1}{(0-\pi)^3} \right)$

$\hookrightarrow -\infty$

$= \infty$, Diverges

(b) y-axis.

(b)



Shells

$V = 2\pi \int_0^{\pi} x \cdot \frac{1}{(x-\pi)^2} dx$

$= 2\pi \lim_{t \to \pi^-} \int_0^t \frac{x}{(x-\pi)^2} dx$

sub $u = x - \pi$, $du = dx$

$\int (u+\pi) u^{-2} du = \ln|u| - \pi u^{-1} + C$

$V = 2\pi \lim_{t \to \pi^-} \left(\ln|x-\pi| - \frac{\pi}{x-\pi} \Big|_0^t \right)$

$= \infty$, DIVERGES

C. Comparison Test for Improper Integrals. Suppose $f(x) \geq g(x) \geq 0$ on the interval $[a,\infty)$. If $\int_a^{\infty} f(x)dx$ converges, then so does $\int_a^{\infty} g(x)dx$. On the other hand, if $\int_a^{\infty} g(x)dx$ diverges, then so does $\int_a^{\infty} f(x)dx$. Sketch:

8. Does $\int_1^{\infty} \frac{\sin^2(x)}{x^2} dx$ converge?

$0 \leq \sin^2(x) \leq 1$

$0 \leq \frac{\sin^2(x)}{x^2} \leq \frac{1}{x^2}$

$\int_1^{\infty} \frac{1}{x^2} dx$ converges by the p-test, so $\int_1^{\infty} \frac{\sin^2(x)}{x^2} dx$ converges by the comparison test.

9. $\int_1^{\infty} \frac{5+\cos(x)}{\sqrt[5]{x^2}} dx$

$\frac{5+\cos(x)}{x^{2/5}} \geq \frac{1}{x^{2/5}}$, $\int_1^{\infty} \frac{1}{x^{2/5}} dx$ diverges by the p-test so $\int_1^{\infty} \frac{5+\cos(x)}{\sqrt[5]{x^2}} dx$ diverges by the comparison test.

D. More exercises. Determine if each of the following integrals converges or diverges. If possible, evaluate the convergent integrals.

10. $\int_0^{\infty} \cos(x)dx = \lim_{t \to \infty} \int_0^t \cos(x)dx = \lim_{t \to \infty} \left(\sin(x) \Big|_0^t \right)$

$= \lim_{t \to \infty} \left(\sin(t) - \sin(0) \right)$ DIVERGES

Oscillates between -1 and $+1$. 0

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.
Study the (a) class notes, (b) text examples, (c) do the text exercises, and (d) do the 4th hour problems.

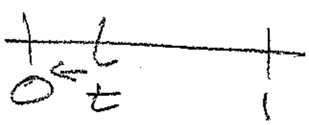
11. $\displaystyle\int_3^\infty \frac{x+3\sqrt{x}}{x}\,dx = \lim_{t\to\infty}\int_3^t \left(1+3x^{-1/2}\right)dx$

$\displaystyle = \lim_{t\to\infty}\left(x+6x^{1/2}\right)\Big|_3^t$

$\displaystyle = \lim_{t\to\infty}\left(t+6\sqrt{t}-3-6\sqrt{3}\right) = \infty$

DIVERGES

12. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\left(\int_t^1 x^{-1/2}\,dx\right)$



$\displaystyle = \lim_{t\to 0^+}\left(2x^{1/2}\right)\Big|_t^1$

$\displaystyle = \lim_{t\to 0^+}\left(2\cdot 1-\underbrace{2\sqrt{t}}_{\to 0}\right) = 2$

13. $\displaystyle\int_0^1 \frac{1-e^{-10x}}{1-e^{-5x}}\,dx$

$\displaystyle = \int_0^1 \frac{\cancel{(1-e^{-5x})}(1+e^{-5x})}{\cancel{(1-e^{-5x})}}\,dx$

$\displaystyle = \int_0^1 (1+e^{-5x})\,dx = \left(x+\frac{e^{-5x}}{-5}\right)\Big|_0^1$

$\displaystyle = 1-\frac{e^{-5}}{5}-\left(0+\frac{1}{-5}\right)$

$\displaystyle = \frac{6}{5}-\frac{1}{5e^5}$