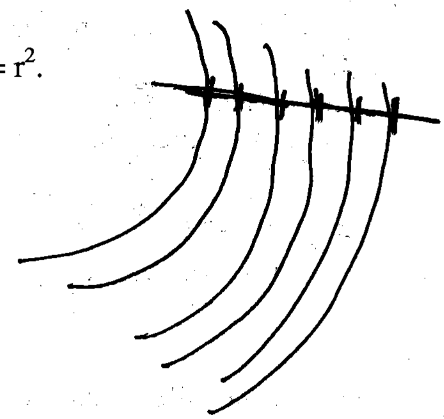
9.4 Orthogonal Trajectories

An **orthogonal trajectory** of a family of curves is a curve that intersects each curve of the family at right angles.

1. Find the orthogonal trajectories of the family of circles $x^2 + y^2 = r^2$. Graph. Observe.



Method:
1. Find the DE describing the family
2. Solve the DE for dy/dx.
3. Write dy/dx for the orthogonal family.
4. Solve the DE

2. Find the orthogonal trajectories of the family $y = ke^{-x}$, $k \neq 0$.

$k = ye^{x}$

tangents $\frac{dy}{dx} = ke^{-x}(-1)$

$= ye^{x}e^{-x}(-1)$

$= -y$

orthog $\frac{dy}{dx} = \frac{1}{y}$

$\int y\,dy = \int dx$

$y^2/2 = x + C_1$

$y^2 = 2x + C$, $C = 2C_1$

3. Find the orthogonal trajectories of the family $y^2 = kx^2$. , $k = \frac{y^2}{x^2}$

$2y\frac{dy}{dx} = k2x$

tangent $\frac{dy}{dx} = \frac{y^2}{x^2}\frac{x}{y} = \frac{y}{x}$

orthog $\frac{dy}{dx} = -\frac{x}{y}$

$\int y\,dy = \int -x\,dx$

$\frac{y^2}{2} = -\frac{x^2}{2} + C_1$

$x^2 + y^2 = C$, $C = 2C_1$

4. Find the orthogonal trajectories to the family of curves $xy = c$, $c \neq 0$.

$y + x\frac{dy}{dx} = 0$

tangent $\frac{dy}{dx} = \frac{-y}{x}$

orthog $\frac{dy}{dx} = \frac{x}{y}$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C_1$

$y^2 = x^2 + C$, $C = 2C_1$

$y^2 - x^2 = C$

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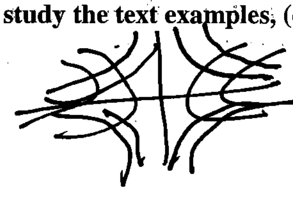
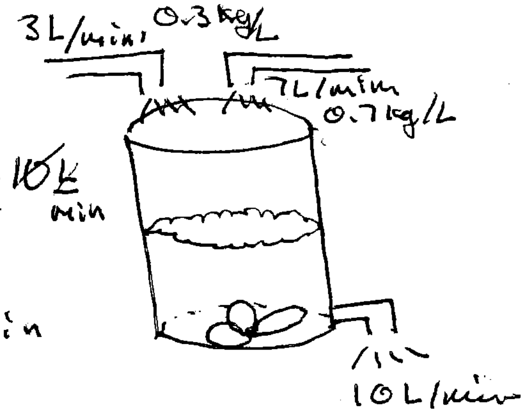
Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.



Mixture problem

A tank which holds 500 L of orange juice mixture is half full, containing water and 2 kg of pure juice. At 9 am two taps open and out of one tap pours 3 L/min of mixture containing 0.3 kg/L of pure juice, while the other tap produces 7 L/min of mixture containing 0.7 kg/L of pure juice. The mixture is well stirred, thoroughly mixed and leaves the tank at the rate of 10L/min.

a) Set up a differential equation to model this activity, letting y be the amount of pure juice in the tank t minutes after 9 am.



$$\frac{dy}{dt} = \text{amount in} - \text{amount out}$$

$$= \frac{3L}{min}(0.3)\frac{kg}{L} + \frac{7L}{min}(0.7)\frac{kg}{L} - \frac{y\,kg}{250L}\,\frac{10L}{min}$$

b) Solve the differential equation.

$$\frac{dy}{dt} = 0.9 + 4.9 - \frac{y}{25} \quad kg/min$$

$$\frac{dy}{dt} = 5.8 - y/25 = 5.8 - 0.04y$$

$$\int \frac{dy}{5.8-0.04y} = \int dt = t + C_1$$

Sub $u = 5.8 - 0.04y$

$du = -0.04dy$

$-25du = dy$

$$\int \frac{-25du}{u}$$

$$= -25\ln|u|$$

$$= -25\ln|5.8-0.04y| = t + C_1$$

$$\ln|5.8-0.04y| = -0.04t + C_2$$

$$|5.8-0.04y| = e^{-0.04t+C_2}$$

$$= e^{-0.04t}e^{C_2}$$

$$= C_3 e^{-0.04t}$$

$$\rightarrow 5.8 - 0.04y = C_4 e^{-0.04t}$$

$$y = \frac{5.8}{0.04} + Ce^{-0.04t}$$

$$y = 145 + Ce^{-0.04t}$$

When $t = 0, y = 2$

$$2 = 145 + C, \quad C = -143$$

$$y = 145 - 143e^{-0.04t}$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

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