7.2 Trigonometric Integrals

$$\boxed{\sin^2(x)+\cos^2(x)=1 \quad \sin^2(x)=1-\cos^2(x)}$$

A. Odd powers of sine and cosine

1. $\int \cos^4(x)\sin^3(x)dx$

$$=\int \cos^4(x)(1-\cos^2(x))\sin(x)dx=\int u^4(1-u^2)(-du)$$

Sub $u=\cos(x)$, $du=-\sin(x)dx$

$$=\int (u^6-u^4)du$$

$$=\frac{u^7}{7}-\frac{u^5}{5}+C$$

$$=\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}+C$$

2. $\int \cos^3(2x)\sin^2(2x)dx$

$$=\int (1-\sin^2(2x))\sin^2(2x)\cos(2x)dx$$

Sub $u=\sin(2x)$

B. Even powers of sine and cosine

$$\boxed{\cos^2(t)=\frac{1+\cos(2t)}{2}, \quad \sin^2(\oplus)=\frac{1-\cos(2\oplus)}{2}}$$

3. $\int \sin^4(3x)dx = \int (\sin^2(3x))^2dx=\int\left(\frac{1-\cos(6x)}{2}\right)^2$

$$=\frac{1}{4}\int\left(1-2\cos(6x)+\cos^2(6x)\right)dx$$

$$=\frac{1}{4}\left(x-2\frac{\sin(6x)}{6}+\int\frac{1+\cos(12x)}{2}dx\right)=\frac{x}{4}-\frac{\sin(6x)}{12}+\frac{1}{8}\left(x+\frac{\sin(12x)}{12}\right)+$$

$$=\frac{3}{8}x-\frac{\sin(6x)}{12}+\frac{\sin(12x)}{96}+C$$

4. $\int \cos^4(5t+7)dt$

$$=\int(\cos^2(5t+7))^2dt$$

$$=\int\left(\frac{1+\cos(10t+14)}{2}\right)^2$$

$$=\frac{1}{4}\int\left(1+2\cos(10t+14)+\cos^2(10t+14)\right)dt$$

$$=\frac{1}{4}\left(t+\frac{2\sin(10t+14)}{10}+\int\frac{1+\cos(20t+28)}{2}dt\right)$$

 Understand the methods so you can solve similar problems.
Understand the concepts so you can solve unfamiliar problems.

Study the (a) class notes, (b) do the 4th hour problems, (c) study the text examples, (d) do the text exercises and (e) read the next text section.

$$=\frac{1}{4}\left(t+\frac{\sin(10t+14)}{5}+\frac{1}{2}\left(t+\frac{\sin(20t+28)}{20}\right)\right)+C$$

$$=\frac{3}{8}t+\frac{\sin(10t+14)}{20}+\frac{\sin(20t+28)}{160}+C$$

7.2

$$\int \tan(t)\,dt = \int \frac{\sin(t)}{\cos(t)}\,dt = -\int \frac{du}{u} = -\ln|u| + C$$

$$u = \cos(t)$$
$$du = -\sin(t)\,dt$$

$$= -1\ln|\cos(t)| + C = \ln|(\cos(t))^{-1}| + C$$

$$= \ln|\sec(t)| + C$$

$$\int \sec(t)\,dt = \int \frac{\sec(t)(\sec(t)+\tan(t))}{\sec(t)+\tan(t)}\,dt$$

$$= \int \frac{\sec^2(t) + \sec(t)\tan(t)}{\sec(t)+\tan(t)}\,du = \int \frac{du}{u}$$

Sub

$$u = \sec(t) + \tan(t)$$
$$du = (\sec(t)\tan(t) + \sec^2(t))\,dt$$

$$= \ln|u| + C = \ln|\sec(t) + \tan(t)| + C$$

$$\sin^2(t) + \cos^2(t) = 1$$

Divide by $\cos^2(t)$

$$\tan^2(t) + 1 = \sec^2(t)$$
$$\tan^2(t) = \sec^2(t) - 1$$

C. Tangent and secant

$$\int \boldsymbol{tan(t)dt = ln|sec(t)| + C}$$

$$\int \boldsymbol{sec(\boxtimes)d\,\boxtimes = ln|sec(\boxtimes) + tan(\boxtimes)| + C}$$

$$\boldsymbol{tan^2(x) = sec^2(x) - 1}$$

5. $\int tan^2(3t)dt$

$= \int (sec^2(3t) - 1)\,dt$

$= \dfrac{tan(3t)}{3} - t + C$

Sub 6. $\int tan^5(3x+1)sec^2(3x+1)dx$ $= \int u^5 \cdot \frac{1}{3}\,du$

Let $u = tan(3x+1)$

$du = sec^2(3x+1)\,3\,dx$

$= \frac{1}{3} \cdot \frac{u^6}{6} + C$

$= \frac{1}{18} tan^6(3x+1) + C$

7. $\int (tan^2(8x) - \sec(3x))dx$

$= \int (sec^2(8x) - 1)\,dx - \frac{1}{3} ln|sec(3x) + tan(3x)| + C$

$= \frac{1}{8} tan(8x) - x - \frac{1}{3} ln|sec(3x) + tan(3x)| + C$

Omit reduction formulas, examples 3, 4, 6,7, 8 and *cos(mx)sin(nx)* and the table of Trig Integrals on p. 410.